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ABSTRACT

The major purpose of this study was 1) to investigate the development of the concept of a unit of measure and the coordination of unit size and the number of units 2) to relate this development to the development of conservation and 3) to determine the role of equivalence and nonequivalence relations in certain conservation and measurement problems. 218 subjects (grades K-2) were individually tested on items selected from a set of conservation and measurement problems. In one set of measurement problems, quantities were measured with different size units of measure. Centering on either unit size or the number of units alone lead to errors similar to those found in conservation problems. In a second set of measurement problems, quantities in different shaped containers were measured with the same unit. Each type of problem was administered in three situations employing different combinations of equivalence and nonequivalence relations. Results are discussed in detail. (Author/MS)

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Technical Report No. 178

THE ROLE OF EQUIVALENCE AND ORDER RELATIONS IN THE DEVELOPMENT AND COORDINATION OF THE CONCEPTS OF UNIT SIZE AND NUMBER OF UNITS IN SELECTED CONSERVATION TYPE MEASUREMENT PROBLEMS

Report from the Project on
Analysis of Mathematics Instruction

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The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to generate new knowledge about the conditions and processes of learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Technical Report is from Phase 2 of the Project on Prototypic Instructional Systems in Elementary Mathematics in Program 2. General objectives of the Program are to establish rationale and strategy for developing instructional systems, to identify sequences of concepts and cognitive skills, to develop assessment procedures for those concepts and skills, to identify or develop instructional materials associated with the concepts and cognitive skills, and to generate new knowledge about instructional procedures. Contributing to the Program objectives, the Mathematics Project, Phase 1, is developing and testing a televised course in arithmetic for Grades 1-6 which provides not only a complete program of instruction for the pupils but also inservice training for teachers. Phase 2 has a long-term goal of providing an individually guided instructional program in elementary mathematics. Preliminary activities include identifying instructional objectives, student activities, teacher activities materials, and assessment procedures for integration into a total mathematics curriculum. The third phase focuses on the development of a computer system for managing individually guided instruction in mathematics and on a later extension of the system's applicability.

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ABSTRACT

This study was designed to investigate the development of certain measurement concepts, to relate this development to the development of conservation, and to determine the role of equivalence and nonequivalence relations in conservation and measurement problems.

218 Ss in grades K--2 were individually tested on 4 to 9 items selected from a set of 18 conservation and measurement problems. The conservation problems were the classical continuous quantity and discrete object problems. In one set of measurement problems, quantities were measured with different size units of measure. Centering on either unit size or the number of units alone lead to errors similar to those found in conservation problems. In a second set of measurement problems, quantities in different shaped containers were measured with the same unit. Each type of problem was administered in three situations employing different combinations of equivalence and nonequivalence relations.

The results of this study indicate that: 1) There is no significant difference in difficulty between conservation and measurement problems due to different combinations of equivalence and nonequivalence relations. 2) Measurement processes are meaningful to most first and second grade students and can be used to compare quantities; however, a significant number of young children readily abandon measurement choices if they are followed by conflicting visual cues; and the majority of first and second grade students do not recognize the importance of a constant unit of measure and incorrectly apply measurement processes that involve more than one unit of measure. 3) Numerical conflict produced by measuring with different units of measure results in the same degree of errors as the visual conflict produced by pouring liquids into different shaped containers in the classical conservation problems. 4) Most conservation and measurement errors result from children centering on an immediate dominant dimension; consequently, problems in which the correct cues appear last are significantly easier than problems in which they are followed by distracting cues. Order, however, is not the only significant variable. Problems in which correct cues are numerical are significantly easier than similar problems in which correct cues are visual.

Chapter I

INTRODUCTION

Statement of the Problem

The major purpose of this study was 1) to investigate the development of the concept of a unit of measure and the coordination of unit size and the number of units 2) to relate this development to the development of conservation and 3) to determine the role of equivalence and nonequivalence relations in certain conservation and measurement problems.

To accomplish these objectives, the study explored the patterns of young children's responses to a collection of different problems in which they were asked to compare two quantities. In each of the problems the Ss were faced with conflicting sets of cues regarding the relationship between the quantities. Several of the problems involved the classical conservation tasks in which two quantities in a comparable state, discrete objects in one-to-one correspondence or quantities of liquid in identical containers, were transformed so that the relation between the quantities was no longer evident; one of the sets of discrete objects was spread out or one of the quantities of liquid was poured into a taller, narrower container. Thus, Ss were faced with conflicting sets of cues as to the relative quantities of objects or liquid. Ss could determine the correct relation by focusing on the original state or by recognizing the compensating relations between the length and density of the discontinuous objects or height and width of the liquid. If they centered on a single dimension in the final state, however, they perceived a different relation between the quantities.

In a second set of similar items the distracting cue was numerical rather than visual. Two quantities of liquid in identical containers were measured using different size units of measure so that to determine the correct relation between the quantities it was necessary to focus on the original state or to recognize the compensating relation between the unit size and number of units. Centering on either unit size or number of units alone lead to the incorrect relation. In a third set of problems the two quantities were measured with the same unit. In one case they were measured into different shaped containers and in another case they started out in different shaped containers and then were measured.

Each type of problem was administered in three situations involving different relations between the stimulus pairs:

1) Equivalence: The quantities were equal and transformed to appear unequal.

2) Nonequivalence I: The quantities were unequal and transformed so that the dominant dimension in both quantities (length or height in conservation, number in measurement) was equal.

3) Nonequivalence II: The two quantities were unequal and transformed so that the direction of the inequality appeared to be reversed.

Differences between items and between individual patterns of responses were examined in an attempt to determine what factors are involved in recognizing the necessity of a constant unit of measure and whether difficulties in coordinating unit size and number of units are specific to the measurement process or are simply an instance of the more general problems of not recognizing the invariance of quantities under various transformations. The following specific questions pertaining to the development of measurement concepts were considered:

1) What differences in young children's responses to items involving quantitative comparisons can be attributed to the type of cue they are confronted with regarding the quantities?

    a) Do distracting visual cues and distracting measurement cues generally lead to the same errors? i.e. What are the differences in responses to the problems in which the quantitative relations are distorted by pouring the liquids into different shaped containers and the problems in which the quantities are measured with different size units of measure?

    b) If there is conflicting evidence on which to base a quantitative judgement, does it matter whether the correct cue is a visual or a measurement cue? i.e. What are the differences in responses to problems in which the quantities can be correctly visually compared but are measured with different units of measure and problems in which the quantities cannot be visually compared but are measured with the same unit of measure?

2) Do young children tend to focus on the type of cue, responding strictly on the basis of either visual or numerical cues; or is the order of the cues the determining factor; or is there an interaction between the two? i.e. Do young children's quantitative errors simply result from responding to the last cue given, or does the type of cue also affect their response?

3) What is the role of coordinating unit size and the number of units in quantitative judgements? When quantities are measured with different units, does the inverse relationship between unit size and the number of units contribute to the correct response; or do those who answer correctly simply ignore the measurement cues and respond on the basis of other sets of cues? Would there be any difference between problems

in which it was possible to distinguish the difference in the size of the units and those in which it was not?

The second major purpose of the study was to determine any differences in the way young children respond to equivalence and nonequivalence relations. Specifically:

1) Are young children's responses to conservation and measurement problems a function of the relation between the quantities being compared? Are Equivalence, Nonequivalence I, and Nonequivalence II problems of equal difficulty?

2) Does assigning a number to quantities differentially reinforce equivalence and nonequivalence relations? For example, does counting the elements of sets in the classical conservation problems identified above have a different effect when the sets have the same number of elements than when they have a different number of elements?

3) If Nonequivalence I is easier than the other relations, is this due to equivalence cues being weaker distractors than nonequivalence cues; or is it due to the fact that in most Nonequivalence I cases the correct relation between quantities can accurately be determined from the distractor cues? Is Nonequivalence I easier in situations where this is not the case, for example, in measurement problems where it is not possible to determine the differences in the size of the units.

A third purpose of the study was to assess the degree to which failure on conservation type tasks can be attributed to experimental procedures.

1) Do _S_s tend to respond to the last choice given to them in the protocols? If asked, "Is there more of one quantity or are they the same?", do they respond "same" because that was the last choice given to them?

2) Does inducing a numerical set by having _S_s count the elements in the array being compared improve performance on conservation problems?

3) To what extent are conservation failures due to _S_s attempting to second guess _E_ and responding to the operation performed on the sets? In other words, if an _S_ is asked to compare two

> quantities and asked to compare them again after thay have been transformed, does he respond that the relation has changed simply because the transformation calls attention to the dimension that has changed and the subject assumes that dimension is what he is being asked to compare? When a liquid is poured from identical containers into different shaped containers, the procedures emphasize the change in shape. Would the same errors occur if the liquid was measured into different shaped containers? In this case the emphasis would be on the measurement rather than on the change in shape.

Mathematical Background

In order to understand how this study relates to the measurement process and to other measurement studies, it is necessary to consider exactly what constitutes the process of measurement.

Measurement can be defined as "the assignment of particular mathematical characteristics to conceptual entities in such a way as to permit (1) an unambiguous mathematical description of every situation involving the entity and (2) the arrangement of all occurrences of it in a quasi-serial order." (Caws, 1959, p.5). Mathematically the process of measurement can be discussed in terms of functions mapping the elements of a domain into some mathematical structure, (usually a subset of the real numbers) in such a way as to preserve the essential characteristics of the domain. (For a more complete treatment of a functional approach to measurement and for the definitions of the mathematical terms used in this discussion, see Blakers, 1967.)

The first requirement for the establishment of a measurement function is to recognize a domain D of elements which possess a given attribute. (The term "elements" is used loosely and can include such things as quantities of a liquid that can be partitioned in an infinite number of ways into distinct elements.) By empirical procedures the domain is given a structure, usually involving the establishment of operations and relations on the objects of the domain. In most common measurement functions this structure is imposed by first establishing a procedure for comparing elements of D on the basis of the given attribute and using this procedure to define an equivalence relation $\sim$ on the elements of D. This equivalence relation is used to partition D into equivalence classes $\tilde{d}$, thereby creating a set $\tilde{D}$ of the equivalence classes of D. The procedure for comparing elements of D also allows one to define an order relation $<$ on D, which turns out to be a strict total order relation on D. i.e. For every two elements d_1 and d_2 of D, exactly one of the following holds: $d_1 \sim d_2$, $d_1 < d_2$, or $d_2 < d_1$. This order relation $<$ yields a corresponding order relation $<$ on $\tilde{D}$ defined as follows: Given $\tilde{d}$ and $\tilde{e}$ in $\tilde{D}$, $\tilde{d} < \tilde{e}$ if $d_1 < e_1$ for any d_1 in $\tilde{d}$ and any e_1 in $\tilde{e}$. Next an operation, $*$, which is both commutative and associative is defined on the set D and extended in a natural way to the set $\tilde{D}$. Thus, $(\tilde{D}, *, <)$ assumes the structure of an ordered abelian semigroup. i.e. For every $\tilde{d}_1$, $\tilde{d}_2$, $\tilde{d}_3$ in $\tilde{D}$, $\tilde{d}_1 < \tilde{d}_2$ implies $\tilde{d}_1 * \tilde{d}_3 < \tilde{d}_2 * \tilde{d}_3$ and $\tilde{d}_3 * \tilde{d}_1 < \tilde{d}_3 * \tilde{d}_2$.

Once D has been given a recognizable structure, the next step is to attempt to define a function μ that maps D into a subset of the real numbers and preserves the essential characteristics of the structure of D. i.e. Given d_1, d_2, d_3 in D:

(1) $\mu(d_1) = \mu(d_2)$ if and only if $d_1 \sim d_2$

(2) $d_3 \sim d_1 * d_2$ implies $\mu(d_3) = \mu(d_1) + \mu(d_2)$ assuming that d_1 and d_2 do not intersect.

Many common measurement functions which measure domains with dense order relations can be defined by arbitrarily selecting a member d_0 of D as a unit. (An order relation is dense if: given d_1, d_2 in D such that $d_1 < d_2$, there exists d_3 in D such that $d_1 < d_3 < d_2$. The length, area, volume, and weight measurement functions measure domains with dense order relations while the counting measure does not.) Then any other element d of D is compared with successive multiples of d_0 until a multiple nd_0 is found such that $nd_0 \lesssim d < (n+1)d$. ($\lesssim$ means $<$ or $\sim$) Next an element d_1 is chosen such that $10d_1 \sim d_0$, and a multiple of d_1 is joined to nd_0 such that $nd_0 * n_1 d_1 \lesssim d < nd_0 * (n_1+1)d_1$. Similarly d_2 and n_2 are chosen such that $nd_0 * n_1 d_1 * n_2 d_2 \lesssim d < nd_0 * n_1 d_1 * (n_2+1)d_2$. Continuing in this manner a decimal number $r = n.n_1 n_2 n_3 \ldots$ is built up and used to define the function

$$\mu : D \to R^+, \text{ by } \mu(d) = r.$$

It is generally possible to define more than one measurement function from D into the real numbers. In the case where the function μ is defined by arbitrarily selecting a member d_0 of D as a unit, a different function μ_1 can be defined by selecting another element d_1 of D that is not equivalent to d_0 and using it to generate μ_1. A natural question to ask is how do the different functions that one might define on D relate to one another? In other words, if μ and μ_1 are measurement functions from $(\tilde{D}, *, <)$ to $(R^+, +, <)$, what is the nature of the automorphism K on $(R^+, +, <)$ such that $\mu = K\mu_1$? The measurement functions used in this study are similarity-invariant measures, which means that the function K is of the form

$$K: x \rightarrow kx \qquad k \varepsilon R^+$$

If $d_0 < d_1$, then $k > 1$; and if $d_0 > d_1$, then $0 < k < 1$. In other words, given two different choices of units d_0 and d_1 and their corresponding measurement functions μ_0 and μ_1, there is an inverse relationship between the relation d_0 and d_1 and the numbers to which any element of D is mapped by μ_0 and μ_1 respectively. In less formal terms, there is an inverse relationship between the unit size and the number of units.

It should be noted that a basic assumption has been made in attributing a structure to D and defining a measurement function from D to R^+. It has been assumed that the attribute that is being measured remains constant under certain transformations and is not affected by the empirical procedures used to define the operations on D. This assumption pervades the entire measurement process, and without it measurement has no meaning. One of the esssential characteristics of a measurement function is that it preserves the relation between the elements of the domain that it measures. If the empirical procedures used to define the function change the relation between the elements of the domain, then the measurement function has no meaning. In fact it makes no sense even to compare elements of D if the process of comparing changes the relation between the elements.

Thus, it would appear that in order to have any meaningful concept of the measurement process it would be necessary to recognize the invariance of certain properties under various transformations, a process that the Swiss psychologist Jean Piaget has termed "conservation." A major goal of this study was to determine how young children's ability to conserve affects certain aspects of their understanding of the measurement process.

A second aspect of the measurement process considered in this study was the order preserving property of the measurement function. The study attempted to assess the degree to which young children recognize that $\mu(d_1) = \mu(d_2)$ if and only if $d_1 \sim d_2$.

A third aspect of the measurement process considered in this study was the role of the choice of the unit in defining the measurement function, the fact that different units define different functions and the nature of the relation between the functions defined by different units. Ss were tested to determine whether they recognized that quantities measured by different functions (i.e. with different units) cannot be compared directly and whether they could find the relation between different measurement functions (i.e. whether they could determine the relative size of the units given the number of each unit in a given quantity).

Significance

Abstract mathematical systems have no necessary logical connection with the real world. The measurement process provides a link of the empirical structures of the real world to the formal structures of mathematics in such a way that empirical properties are carried over into the corresponding mathematical system.

In recognition of the fundamental nature of the measurement process in mathematics, the study of measurement is becoming an established part of the mathematics curriculum of the early primary grades. In a survey of 39 completed or partially completed elementary mathematics series, Paige and Jennings (1967) found that by the end of the first grade about half of the texts had introduced linear and liquid measurement.

Some current proposals advocate an even more extensive treatment of measurement concepts. The Cambridge Conference on School Mathematics (1963) and the K--13 Geometry Committee (1967) have advocated teaching measurement of length, area, and volume using both arbitrary and standard units in the early primary grades.

The mathematics program Developing Mathematical Processes (DMP), being developed at the University of Wisconsin Research and Development Center for Cognitive Learning, has made measurement processes the basis for developing fundamental number concepts (Romberg, Fletcher, & Scott, 1968). In grade I the arithmetic units include activities in which children choose arbitrary units of length and weight and use them to measure specified objects (Romberg & Harvey, 1969, 1970).

The proposed Geometry unit for grades K--2 is to include measuring and comparing length, area, volume, weight, and time using both arbitrary and standard units of measure (Harvey, Meyer, Romberg, &

Fletcher, 1969).

The conservation and measurement research of Piaget (1952, 1960) seems to raise serious questions as to the advisability of expecting students in the early primary grades to master these measurement concepts. Huntington (1970) applied Piaget's (1960) studies and Beilin (1971) and Lovell's (1971b) hypothesis that mathematical ideas which depend on a level of logical thought beyond a child's capacity are only partially or very tenuously learned to the SMSG program for grades 1--3 and concluded that "Piaget's studies indicate that the instruction on linear measurement is placed too early and is instructed too narrowly." (p. 232) This conclusion is not unreasonable in light of Piaget's findings that 8--8½ years is the age that a significant proportion (75%) of children first master the easiest measurement process (length).

Other mathematicians and psychologists (The Cambridge Conference, 1963 and Sullivan, 1967) have challenged the direct application of Piaget's conclusions to determine the sequence of subject matter in the curriculum. Sullivan (1967) states:

> The Piagetian contribution to the structure and sequencing of subject matter is more apparent than real. This is clearly not the fault of Piaget, but rather of his educational followers. Uncritical extrapolation of Piaget's observations and his methodological considerations (e.g., logico-mathematical model) is, in the opinion of the present author, harmful to the advancement of educational knowledge. The use of Piaget's stages as indicators of "learning readiness" seems most premature and needs more careful consideration on both the research and theoretical levels. (p. 33)

Even those who question the validity of using Piaget's conclusions as the final arbitrator in determining the scope and sequence of materials in the curriculum concede that his studies have definite implications for curriculum development and the assessment of learning outcomes. This point of view is clearly expressed by Lovell (1971a):

> These results [the studies of Piaget and his followers] do not, of course, imply that all work on measurement of length [or any other measurement] should be postponed until the child fully understands what he is doing. Rather the experiences derived from activities involving measurement, which the child carries out at the teacher's suggestion, provide the basis out of which understanding arises with the growth of thinking skills. The important point for the teacher is to realize to what extent a child is carrying out measurement operationally--that is, with full understanding--or to what extent it is being carried out in rote fashion. (pp. 104-105)

Measurement concepts are going to be taught. The goal of this study was to attain a clearer picture of some aspects of young children's intuitive understanding of measurement concepts so that activities can be constructed to help children overcome basic misconceptions, and evaluation items can be designed to assess the success in dealing with these misconceptions.

Just as measurement processes are a basic component of the DMP mathematics program, so too are order and equivalence relations. Equivalence relations have traditionally played a much greater role than order relations in the mathematics curriculum of the primary grades. Following the recommendations of The Cambridge Conference (1963), DMP has integrated equivalence and order relations from the first. A major goal of the proposed study was to determine if any differences exist in young children's ability to deal with equivalence and order relations in conservation and measurement problems.

> For the psychologist and educator, the crucial problem... is to secure an adequate picture of the transitions that occur in the sequence [of the development of conservation], and eventually to specify the factors involved in them. (Almy, 1966, p. 19)

> ...little is known thus far about the specific ways in which the transition from lack of conservation [or measurement] to the presence of conservation [or measurement] takes place. It is apparent, however, that an adequate explanation of this problem ultimately requires a clearer understanding of the psychological processes at work in this transition stage. ...a more detailed examination of the interrelationship among different tasks involving conservation and closely related concepts should likewise extend our understanding of this problem.
> (Wohlwill & Lowe, 1962, p. 153)

This study examined the interrelationship among a number of different tasks involving conservation and measurement concepts and different combinations of equivalence and order relations. It attempted to generalize the development of conservation to include problems involving numerical as well as visual distractors and to determine the role of equivalence and order relations in the transition from lack of conservation to conservation.

Chapter II

RELATED RESEARCH

The motivation for this study was derived primarily from the work of two groups of researchers, the Swiss psychologist Jean Piaget and his followers and the Soviet educator-psychologists P. Ya.Gal'perin and L.S. Georgiev. In fact, the current study can be viewed in part as an attempt to resolve their conflicting views.

Piaget's Conservation and Measurement Studies

Piaget has extensively studied young children's conception of quantity and measurement in a variety of situations. For Piaget the central idea "underlying all measurement is the notion that an object remains constant in size throughout any change in position." (Piaget, Inhelder, & Szeminska, 1960, p.90) In his now classic conservation studies Piaget demonstrated that young children do not realize the invariance of most quantitative properties under various transformations.

Piaget (1952) proposed three stages that a child passes through in the attainment of conservation. In stage I "quantification is restricted to the immediate perceptual (unidimensional) relationships." (Piaget, 1952, p.11) For example, given two equivalent containers (A1, A2) containing the same quantity of liquid, if A2 is poured into a taller, narrower container (B2), a child in this stage asserts that there is more liquid in the taller container, focusing strictly on the height and ignoring the transformation from the containers in which he agreed the quantities were equal. If the quantities are such that the height in A1 and B2 is the same, the child asserts that the quantities are equal, even though B2 is narrower.

In stage II the child is capable of assuming the quantity is not changed when it is transformed, as long as the transformation is not too great. Children at this stage begin to coordinate different dimensions; and although they are not entirely successful, they are not wedded to a single dimension and will not, for example, assert that there are equal quantities of liquid in different containers just because the liquids are the same height.

In stage III the child successfully conserves irrespective of the nature of the transformations.

The significance of the conservation concept comes from the diversity of situations to which it applies. Piaget (1952) exposed young children to a number of different tasks involving quantitative comparisons and found stages of development paralleling those described above. Several of these tasks are worth mentioning here because they are similar to items in the current investigation.

In one problem _Ss_ were shown a row of objects and asked to pick out a number of objects equal to it. First stage _Ss_ made rows of the same length but of different density. Second stage _Ss_ placed the objects in one-to-one correspondence, but thought the equivalence ceased to exist when one of the rows was spread out. In the third stage, equivalence persisted under distortion.

In another problem Ss dropped beads one by one into a container at the same time E dropped them into a different width container. Thus, Ss were faced with conflicting cues in terms of the one-to-one correspondence of the beads and the different levels of the beads in the containers. Piaget found the development for this task parallels the development of conservation of liquids.

Based on studies of length, area, and volume Piaget et al. (1960) proposed a similar stagewise development of measurement, which is interrelated with the development of conservation. Piaget's measurement problems can be divided into two broad classes which correspond to the two major divisions within the mathematical definition of measurement described above. In the first class of problems, objects of the domain D were compared directly on the basis of a given attribute without assigning a number to the attribute. The conservation studies described above fall into this class. In the second class of problems, measurement functions were applied and attributes were measured with different units of measure.

In studying the child's concept of length, Piaget found the same stages of development that he identified for conservation; however, he also found that distinct substages, A and B, exist in stages II and III. In the earliest stages, I and IIA, length is primarily a function of endpoints. Children in these stages of development generally ignore undulations or angles in objects being compared and base their judgements on the positions of the endpoints. In one task Ss were asked to assure themselves that two strips of cardboard were of the same length. Then one of the strips of cardboard was cut into several pieces and arranged in various configurations. Ss at this stage of development generally asserted that the uncut strip was longer. Occasionally Ss in this stage focused on the number of angles, especially in cases where this factor was exaggerated by having a large number of angles. Similarly Ss who had agreed that two segments were the same length when their endpoints were aligned thought the segments no longer were the same length if they were moved to a staggered position or were moved so that they were at an angle.

In Stage IIB children oscillate between conservation and nonconservation responses or arrive at conservation through a lengthy process of trial and error. In stage III conservation is immediate.

In studying the development of the measurement functions, Piaget asked Ss to judge the relative lengths of strips of paper mounted on cardboard sheets in a variety of linear arrangements involving right angles, acute angles, etc. After the S had replied, he was given a number of movable strips and asked to verify his judgement. Later he was given short strips of cardboard--3 cm., 6cm., and 9 cm. long--and asked to measure the mounted strips.

The earliest stages, I and IIA, are characterized by a "wide variety of responses which have only negative characteristics in common." (Piaget et al., 1960, p.117) Children do not conserve length and are totally incapable of using a unit of measure. They generally rely on visual comparisons and have no confidence in measurement. When asked to measure, some children simply run the unit along the line, making no subdivisions into equal units. Others only cover part of the line or partition it into unequal sections. No child in this stage realizes the importance of a constant size for the unit of measure.

In substage IIB conservation is dimly perceived, and children begin to understand the use of a unit in measuring. By trial and error children gradually discover that if it takes more units to cover A than to cover B then A is longer than B; however, children fail to recognize the importance of the size of the unit and often count a fraction of a unit as a whole or equate two lines that measure the same number of units with different size units of measure.

In substage IIIA, conservation and the use of a common unit are both immediate; but children continue to ignore the size and completeness of units of measure. In substage IIIB children successfully conserve and measure. They recognize the importance of the size of different units of measure and understand the inverse relationship between unit size and number of units.

Piaget found similar stages of development for area and volume. In studying the development of the concept of area, Piaget asked Ss to compare the size of two identical rectangles made up of six squares each, arranged in a two by three configuration. After an S agreed that the two rectangles were the same size, the squares in one of the rectangles were moved around to create a different shaped region; and the S was asked to compare the size of this region with the size of the undistorted rectangle.

Two techniques were used to study the development of area measurement functions. Both involved comparing different shaped figures by measuring with different units. In the first type of problem Ss were given enough units to cover the figures, but the units were of different sizes and shapes. Some were squares, some were rectangles (two squares), and some were triangles (squares cut diagonally in half). In the second type of problem Ss were given a limited number of square unit cards, which they had to move by successive iteration from one part of the region being measured to the next. Some regions were shaped in such a way that it was impossible to cover them with the given units without intersecting the exterior of the region. Thus, it was necessary for Ss to consider fractions of units.

Based on these and several other related studies, Piaget et al. (1960) concluded that the development of the concept of area "is identical with that found to govern the conservation and measurement of lengths in one or two dimensions." (p. 274)

> Even at level IIA (as at level I) children confine themselves to perceptual judgements and the areas are not conserved when their appearance is modified. Again, they cannot measure areas because they lack conservation of the moving middle term so that there is no transitivity. Children at level IIB gradually come to make a number of true judgements, but their success is the product of intuitive adjustments and so is lacking in generality. Likewise we now find some degree of transitivity, so that measurement as such begins to be seen at level IIB, but

> to this there are many limitations. At level IIIA there is operational conservation of areas when their shape is altered (although the conservation is limited to the area enclosed by a given perimeter and does not extend to the complementary area outside). Middle terms now serve as common measures because congruence is recognized as a transitive relation. But children still fail to understand the concept of a unit so that when calculating the total extent of an area they count all its parts as equivalent regardless of their size. Finally, at level IIIB conservation is generalized to cover complementary areas and this level marks the beginning of true measurement, involving unit iteration. (Piaget et al., 1960, pp. 274-275)

In Piaget's volume studies _S_s were shown a solid block 4 cm. high with a square base 3 cm. x 3 cm. and told that this block was a house built on an island 3 cm. x 3 cm. Using 1 cm. blocks, _S_s were asked to build houses that had "as much room" as the original house on other islands of different shape and area. In another problem _S_s were asked if the height of water in a container would change if the configuration of a set of metal cubes that was submerged in the water was altered.

Stage I children are unable to relate to these problems. By stage IIA children can understand the problems but cannot conserve volume and cannot make any comparisons between solids beyond one-dimensional relations. Therefore they ignore the change in the base and construct their house the same height as the model. In stage IIB children attempt to compensate for the smaller base by increasing the height of their house, but they do not use any common measures and generally do not increase the height sufficiently to compensate for the decrease in volume. In stage IIIA children conserve interior volume but not displaced volume. In other words they recognize that the volume of a solid does not change when its shape is altered but do not recognize that if the solid is submerged in water the height of the water is the same before and after the change in shape. Children continue to inadequately compensate for the decrease in the area of the base of their house. Responses in stage IIIB are similar to those in IIIA. Children begin to correctly measure the dimensions of the solids but are unable to establish the relation between lengths and volumes. Finally in stage IV, children discover that volumes are equal if and only if the product of their respective dimensions are equal. In this stage children also begin to conserve displaced volume.

Thus, it is Piaget's view that the development of measurement and conservation concepts are integrally related and that the same general pattern of development persists across all types of measurement operations. In general, the earliest stages in the development of measurement concepts, I and IIA, are characterized by a dependence on one-dimensional perceptual judgements. Transformations from prior states are completely ignored, and quantities are compared on the basis of a single dominant dimension. In stage IIB children begin to make a number of true judgements as long as distortions in quantities being compared are not too great. Correct judgements in this stage are largely a result of trial and error, and at best, children in this stage have a dim concept of conservation and some notion that greater quantities measure more units. In stage IIIA children begin to conserve and measure using a common unit; however, they fail to recognize the importance of a constant unit of measure. In stage IIIB children successfully conserve and measure. They recognize the importance of different units of measure and understand the inverse relationship between unit size and number of units. It is not until stage IV, however, that children finally discover the mathematical relation between area and volume and their respective dimensions.

Piaget's conservation research has been a popular target for further investigation. A number of studies--e.g. Dodwell (1960, 1961), Wohlwill (1960), and Hyde (1959)--have been conducted replicating Piaget's procedures and have, on the whole, supported his description of the development of conservation. Research on measurement concepts has not been so popular. A systematic search of the literature uncovered only four studies that attempted to verify Piaget's proposed stagewise development of measurement. Studies by Lovell, Healey, and Rowland (1962), Lovell and Ogilvie (1961), and Lunzer (1960) generally supported Piaget's conclusions, whereas Beilin and Franklin (1961) found evidence that conservation of one-, two-, and three-dimensional entities are achieved in that order, rather than developing simultaneously as Piaget had suggested.

Whereas Piaget takes the view that conservation precedes measurement, Bearison (1969) used measurement operations to teach children to conserve. Nonconservers were provided with experiences in which they compared two quantities of liquid in terms of the number of identical beakers containing the two quantities. As soon as the Ss mastered the numerical comparisons, first one set then both sets of beakers were transferred to larger beakers; and the Ss were again asked to compare the quantities of liquid. The final phase of training simply involved the standard liquid conservation task. Bearison concluded that:

> The effects of training facilitated the conservation of continuous quantity and transferred to the conservation of area, mass, quantity, number, and length. The explanations offered for conservation by the trained conservers were identical to those elicited from a group of "natural" conservers, and the effects of training were maintained over a 7-month period. (p. 653)

Gal'perin and Georgiev's Study

Whereas Piaget's research has covered the entire range of the measurement process, focusing to some degree on the basic assumption that the properties being measured remain invariant under transformation--two Soviet researchers P. Ya. Gal'perin & L. S. Georgiev (1969) have concentrated their efforts on the definition of the measurement function, especially on the role of the unit. They administered a series of measurement problems to 60 students from the "upper group" of a Soviet kindergarten (6 yrs., 6 mos. to 7 yrs., 2 mos.) from which they concluded that young children taught by traditional methods lack a basic understanding of a unit of measure, that they are indifferent to the size and fullness of a unit of measure and have more faith in direct visual comparison of quantities than in measurement by a given unit. These conclusion were based on the following items:

A) Indifference to the size of the unit of measure: Ss were given two identical cups filled with rice and asked to measure the rice in one of the cups into separate piles using a teaspoon, and then using a tablespoon, to measure the rice in the other cup into another group of piles. Ss were then asked which group of piles contained more rice. Fifty Ss responded that there was more rice in the teaspoon group where there were more piles.

B) Indifference to the fullness of the unit of measure: Having put five spoonfuls of rice on the table and taken four away, Ss were asked how many spoonfuls were left. Thirty-two Ss responded incorrectly due to the fact that they had not checked the fullness of the spoons. Eighteen indicated that there was some number greater that one left, and 10 indicated that there was one spoonful left even though there was a great deal more left.

C) Visual comparison vs. measurement: Ss were asked whether there was more rice in a pile where the Ss had just placed four spoonfuls of rice (pile I) or in a pile in which they had just placed two spoonfuls of rice which had been spread out by E (pile II). They were then asked whether there was more rice in pile II or in a third pile which also contained two spoonfuls but which had not been spread out. Twenty-eight Ss chose pile II as the largest of the three. Seventeen recognized that pile I was larger than pile II but did not realize that pile II and pile III contained the same amount of rice.

On the basis of this study Gal'perin and Georgiev (1969) devised a program of 68 lessons that specifically dealt with measurement concepts and systematically differentiated between units of measure and separate entities. During the 1959-1960 school year, this program was administered to 50 children from the "upper group" of the same kindergarten used in the first investigation. Pretest scores were similar to those reported above; however, on the posttest at least 96% of the Ss correctly responded to each of the above questions.

Carpenter (in press) administered the measurement test items to a group of 20 American first graders and obtained results similar to those obtained for Soviet kindergarten children who had not studied the Gal'perin and Georgiev program. Eighteen of the 20 Ss missed Item A, in which they were asked to measure equal quantities with different size spoons. Sixteen said there was more rice where there were more spoonfuls and two thought there was more rice where the piles were larger.

Eighteen Ss missed Item B, in which they were asked to measure out five spoonfuls of rice and then measure back four. Twelve indicated that there was more than one spoonful left. Five of these said that there were more than five spoonfuls. Six Ss responded that there was only one spoonful left even though there was actually much more than one spoonful. None of these six could explain why they could not get all the remaining rice in the spoon.

Fourteen Ss missed Item C, in which they were asked to compare the three piles of rice. Only 6 missed the first part, but 14 insisted that there was more rice in the second pile than in the third. Of the 8 Ss who answered part one correctly but missed part two, 6 said there was more in pile II than in pile III because pile II was spread out; but there was more in pile I than in pile II because there were four spoonfuls in pile I and only two in pile II. On the same question these Ss changed the basis for their responses, one time responding on the basis of how the piles looked and one time responding on the number of spoonfuls of rice in the piles.

Carpenter also administered some additional items in an attempt to discover the basis for the errors in the Gal'perin and Georgiev problems. In order to gain further insight as to the basis for the responses in the item comparing three piles of rice, part of it was run backwards. (Item D) Four spoonfuls of rice were poured on the table from a cup. Next to this pile two spoonfuls of rice were poured and spread out. Ss were asked which pile contained more rice. If S responded that the spread out pile contained more rice, the rice in both piles was measured into two identical opaque cups; and the S was asked which cup contained more rice. All but one S who correctly compared the piles containing two and four spoonfuls when they were measured out could visually distinguish the larger pile even when they were not measured, and two Ss who could not correctly compare the two piles when they were measured could when they were not measured. All of the five Ss who could not visually distinguish which pile was larger said there was more rice in the cup containing four spoonfuls after the rice was measured. Thus, although many Ss relied on visual comparison when the visual stimuli followed the measurement stimuli, none did when the order was reversed.

In another item (Item E) Ss measured equal quantities of rice with different size spoons. It differed from Item A in that the rice was measured into opaque cups rather than into individual piles. Item E was almost as difficult as Item A. Of the 18 Ss who missed Item A, 16 also missed Item E and no S got Item E and not Item A. Whereas only 2 Ss chose the quantity of rice measured with the larger spoon as having more rice in Item A, 7 did in Item E. Another item (Item F) was administered that was similar to Item E except that the original quantities of rice were not equal but measured the same number of spoonfuls when measured with different spoons. Only 3 of the 20 Ss missed this item. Of the 17 who answered correctly, 8 explained their answer in terms of the size of the spoons, 7 gave conservation type explanations, and 2 were not able to explain their response.

Although American children's performance on the Soviet items was similar to the performance of Soviet children found by Gal'perin and Georgiev, the results on the new items and a different interpretation of the results on the Soviet items yielded a different set of conclusions:

1) Young children are not indifferent to the size of units of measure; but just as in Piaget's conservation problems, they are only

capable of making one-dimensional comparisons. Thus, in Items A and E most Ss centered on a dominant dimension, generally the number of units; but in Item F they correctly noted the significance of the different units.

2) Young children do not rely primarily on the visual comparisons as Gal'perin and Georgiev concluded, but rather they respond on the basis of the last stimulus available, be it numerical or visual. In Items A and E, in which equal quantities were measured with different size spoons, most Ss completely ignored the correct visual cues and responded on the basis of numerical distractors; and in Item D, Ss correctly responded on the basis of numerical cues when the numerical cues came last.

3) The research by Piaget (1952, 1960) cited above also tends to make less credible Gal'perin and Georgiev's hypothesis that improper application of the unit of measure is the source of measurement errors. As demonstrated above in conclusions 1 and 2, the Soviet results are perfectly consistent with Piaget's theories; however, it is difficult to see how most conservation errors could be attributed to errors in young children's concept of a unit of measure.

4) There is tentative evidence that the type of relation between quantities being compared may affect the results in some measurement and conservation problems. Item F, in which Ss started with unequal quantities which measured the same number of spoonfuls, was significantly easier than Item E, in which Ss started with equal quantities which measured different numbers of spoonfuls. Both Carpenter and

Gal'perin and Georgiev found that in Item C the task comparing two spoonfuls of rice with four spoonfuls of rice was easier than the task comparing the two piles of two spoonfuls each. The results of this last item are somewhat mitigated by the fact that in Item D it was discovered that most Ss could visually distinguish the difference between the pile containing four spoonfuls and the spread out pile containing two spoonfuls; however, six of the eight Ss who correctly answered the inequality part of Item C but missed the equality part based their correct responses on the number of spoonfuls of rice and not the visual difference in piles.

The Role of Relations in Conservation Problems

Piaget has not been concerned with differences in the relations between sets in his conservation and measurement studies. He has been attempting to assess the child's conception of number, length, weight, etc. of a single quantity and has simply used equivalent sets as an experimental convenience. Elkind (1967), Van Engen (1971), and Wohlwill and Lowe (1962) have questioned Piaget's procedure of using what Elkind (1967) calls "conservation of equivalence" tasks to assess conservation of number, length, weight, etc. For example, Elkind (1967) hypothesized that "identity conservation" (invariance of a quantitative attribute--e.g. numerousness, weight, volume--under a reversible transformation) precedes equivalence conservation. This hypothesis has been supported in studies by Hooper (1969) and McMannis (1969) while a study by Northman and Gruen (1970) found no differences between the two types of conservation. Almy (1966) and Greco, Grize, Papert, and Piaget (1960) found that "conservation of number" (invariance of the number assigned to a set of discontinuous elements under a reversible transformation) also precedes conservation of equivalence.

In a study involving 98 first grade boys, Zimiles (1966) found no difference in difficulty between conservation tasks using equivalent sets of discrete objects and conservation tasks using nonequivalent sets of discrete objects (Nonequivalence II). Each S was individually tested on eight items that varied on the following constraints:

1) Relation between sets: Equivalence or Nonequivalence II.
2) Significance of conservation materials: Toy trucks or blocks.
3) Homogeneity of conservation materials: All the trucks or blocks were the same or they varied in size and color.
4) *Size of arrays: Small, three objects in each row in the* Equivalence condition and rows of three and five objects in the Nonequivalence II condition; or large, seven objects in each row in the equivalence condition and rows of seven and nine objects in the Nonequivalence II condition.

Each Equivalence item was paired with a corresponding Nonequivalence II item that was identical on the other three constraints. The order of items was varied between Ss to offset any order effect.

There was no significant difference in difficulty between the Equivalence and Nonequivalence II tasks for either the large or small array problems. Over all tasks the per cent of correct responses for Equivalence and Nonequivalence II conditions was 64% and 59% respectively for the small condition and 56% and 54% respectively for the large condition.

Since different Ss received different first problems, it was possible to analyze Equivalence and Nonequivalence II differences by examining Ss' responses to the first items they received. This analysis, which eliminated any possibility of item interaction, also yielded no significant differences in difficulty between Equivalence and Nonequivalence II tasks.

Although the Equivalence and Nonequivalence II items appeared to be of equal difficulty, there was evidence that a substantial amount of individual Ss' inconsistency of performance between items could be attributed to differences in Equivalence and Nonequivalence II conditions.

Several other studies have administered both equivalence and nonequivalence items; and although these studies were not designed to test for differences between the two types of tasks, their results can be examined to determine whether differences did in fact exist. Analysis of individual items in studies by Carey and Steffe (1968) and Harper and Steffe (1968) indicated no clear cut differences in difficulty between Equivalence and Nonequivalence II items. On the other hand, in a study of conservation of discontinuous quantity with children age 2 yrs., 3 mos. to 3 yrs., 10 mos., Piaget (1968) found a significantly greater number of correct answers in nonequivalence situations. Beilin (1968) and Rothenberg (1969) also reported significantly more correct answers to problems in which the relations between sets was nonequivalence; however, their tasks were not traditional conservation problems, and experimental variables appeared to favor the nonequivalence situations.

There is other research which, although it did not directly involve conservation problems, supports the contention that young children respond differently to equivalence and nonequivalence relations. Halford (1968, 1969, 1970) found that even nonconservers could use the information from the standard liquid equivalence conservation task to compare the untransformed container of liquid to containers the same diameter but of different height than the container into which the equivalent quantity of liquid was poured. He concluded that children learn to classify equal containers by first learning to classify unequal containers.

In a study in which 76 Ss ranging from age 5 yrs., 5 mos. to adults were asked to judge whether different pictures were of the same person or were of different people, Saltz and Sigel (1967) found that young children tend to overdiscriminate and find differences where differences do not exist and that this tendency decreases with increasing age.

Griffiths, Shantz, and Sigel (1967) found that preschoolers have more difficulty using the word "same" than the words "more" or "less" when comparing the length and weight of objects, but there was virtually no difference in their use of these words when comparing objects on the basis of number.

In a study of 316 children in grades K--4 in which Ss were asked to compare the area of different regions made up of identical small squares, Beilin (1964) found that problems that required judgements of equality were more difficult than problems that required judgements of inequality. Similar differences in young children's ability to compare objects on the basis of length were found by Carey and Steffe (1968).

Smedslund (1966) found that young children fail to maintain choices of equality. Forty per cent of kindergarteners and 21% of the first graders tested abandoned a decision that two squares were equal in area immediately after they chose them to be equal. Similar results were found by Fleishmann, Gilmore, and Ginsberg (1966). Three of 15 Ss (20%), whose ages ranged from 5 yrs., 5 mos. to 6 yrs., 4 mos., failed a conservation of equivalence task in which beads were poured from one of two identical glasses into a third identical glass. As in the Smedslund study, these Ss failed to maintain their equality judgement even though there was no visual conflict introduced by the transformation.

In a study of 32 preschoolers, Uprichard (1970) found that treatments in which Ss learned to classify sets on the basis of equivalence were mastered more quickly than treatments in which Ss classified sets on the basis of "greater than" or "less than," and learning sequences that began with equivalence were more effective than sequences that began with either "greater than" or "less than."

Discussion

Thus, the results of past research indicate that differences in the relation between the sets being compared in conservation problems do not produce differences in responses that are consistent across all conservation problems. In the most systematic study of the role of equivalence and nonequivalence relations in conservation problems, Zimiles (1966) found no differences for tasks using discrete objects. Carpenter (in press) on the other hand found clear differences in difficulty between tasks comparing equivalent quantities and tasks comparing nonequivalent quantities. Several factors could have accounted for the different results of the two studies:

1) The problems in the Carpenter study utilized continuous quantities whereas Zimiles used discrete objects. Comparisons of discrete objects arranged in one-to-one correspondence are relatively straightforward and precise while equalizing liquid quantities in identical containers is much more difficult and involves a degree of estimation. Smedslund (1966) and Fleishmann et al.(1966) demonstrated that young children readily abandon choices of equivalence. Perhaps some failures in liquid conservation of Equivalence problems are the result of Ss being unsure of their original choices of equivalence and therefore abandoning them. In the liquid nonequivalence problems and all discrete object problems, Ss are not faced with the same difficulty and, therefore, do not so readily abandon their judgements of the relation between the quantities.

Continuous quantity-discrete object differences could also be accounted for in terms of Festinger's (1957) theory of cognitive dissonance. If in the liquid Equivalence task Ss are unsure of the initial equivalence but required to make judgements of equivalence, dissonance is created. When the liquids are poured into different shaped containers, thereby creating the illusion of nonequivalence, Ss attempt to reduce dissonance by abandoning their equivalence choice. In the liquid nonequivalence problems and all discrete object problems, the original choice is straightforward and therefore creates no dissonance. Thus, there is no dissonance to reduce; and Ss do not so readily abandon their original judgements.

2) In the Zimiles study the nonequivalence task was of the Nonequivalence II type while in the Carpenter study one of the nonequivalence tasks was Nonequivalence I, and the results of the other are suspect due to Ss' ability to visually distinguish the differences. Logically one would expect Nonequivalence I tasks to be easier than the other two since accurate judgements can be made from the final state of the quantities by simply comparing them on the dimension that differs. To make similar judgements in Equivalence and Nonequivalence II tasks, it is necessary to know the degree of change in one dimension required to offset the inverse change in the other dimension. Even adults cannot accurately evaluate the final states of Equivalence and Nonequivalence II problems without seeing the transformation. Thus, it may be that the relation between sets does not affect the difficulty of conservation and measurement problems; but Nonequivalence I problems are easier simply because they do not require true conservation.

3) In the Zimiles study all comparisons were visual whereas in the

Carpenter study measurement operations entered in. Possibly equivalence-nonequivalence differences resulted from differences in Ss' ability to handle equality and inequality relations with numbers.

Finally, Zimiles' study should not be taken as conclusive proof that there are no differences in the development of Equivalence and Nonequivalence II, even for discrete objects. The studies of Griffiths et al. (1967), Piaget (1968), and Saltz and Sigel (1967) indicated that equivalence-nonequivalence differences may be more pronounced for younger children.

Chapter III

PURPOSE AND PROCEDURES

Purpose

The purpose of this study was to systematically investigate and extend some of the hypotheses generated by the Carpenter (in press) study regarding the development of measurement concepts. Carpenter reached a set of tentative conclusions contrasting the theories of Jean Piaget and P. Ya. Gal'perin and L.S. Georgiev on the development of measurement concepts. These conclusions were based on a limited set of problems and a small sample of Ss and are best called hypotheses rather than conclusions. The current investigation systematically tested these hypotheses by methodically varying the constraints upon which they were based. Some of the questions regarding the development of measurement concepts that the Carpenter study and other measurement and conservation studies have left unanswered or only partially answered are:

1) How closely do the measurement errors like those in the Carpenter study correspond to classical conservation errors? Do measurement problems where the distracting cues are numerical produce the same degree of errors as the corresponding conservation problems in which the distracting cues are visual? How do the errors on both of these types of problems relate to responses on measurement problems in which the correct cues are numerical and the distracting cues are visual?

2) What role does assigning numbers play in any possible differences between measurement and classical conservation problems? Bearison (1969) and Zimiles (1963) have both suggested that assigning numbers to quantities in conservation would improve conservation performance. If there

are differences between measurement and conservation problems, do they simply result from inducing a numerical set as Zimiles and Bearison have suggested? What effect would assigning numbers to quantities have if the numbers were just supplementary information and were neither necessary to determine the correct relation between quantities nor dominant distracting cues? For example, how would counting the elements in the standard conservation of discontinuous quantity problem affect responses?

3) Is the order of cues the only significant factor in conservation and measurement errors, or does the type of cue, numerical or visual, affect responses?

4) Carpenter found that in measurement problems using different units of measure some _S_s centered on the unit size and others centered on the number of units. How would not being able to distinguish the correct relation between units affect responses to measurement problems? How important is it to be able to perceive the inverse relation between unit size and number of units in order to conserve the correct quantitative relation between quantities?

Carpenter also found that variations in the relations between quantities being measured and compared provided different insights into the development of measurement concepts. He found significant differences between measurement problems that involved equivalence relations and similar problems that involved nonequivalence relations. Over four times as many _S_s correctly answered the problem in which nonequivalent quantities of rice measured the same number of spoonfuls as correctly answered the corresponding problem in which equivalent quantities measured different numbers of spoonfuls. Were these differences due to distinct abilities to conserve equivalence and nonequivalence relations, or were they simply due to the fact that in the nonequivalence problem the correct relation between quantities was discernible from the final state of the quantities and true conservation was not required? This later hypothesis would imply that young children actually understand the effect of variations in unit size but are unable to coordinate these variations with the inverse variations in the number of units in equivalence problems. Would the same differences exist between equivalence problems and nonequivalence problems in which it was not possible to simply distinguish these differences? i.e. Would the same differences exist between Equivalence and Nonequivalence II problems or between Equivalence and Nonequivalence I problems in which the correct relation between units was not discernible?

If there are differences between problems due to equivalence and nonequivalence relations, how general are they and what causes them? Do the same differences exist across all conservation and measurement problems, or do they just occur in specific instances? What role do number and measurement concepts play in any possible variations?

Specific Items

In order to investigate these questions the following items were administered to young children in grades K--2:

I. Conservation of Discontinuous Quantity

   A. Equivalence: The S was shown two rows of seven blocks each arranged in one-to-one correspondence and asked to compare the number of blocks in the two rows. Then one of the rows was spread out so that it was about twice as long as the other row, and the S was again asked to compare the number of blocks in the two rows. (See Figure 1)

B. Nonequivalence I: The S was shown two rows of blocks. One row contained seven blocks; and the other contained nine blocks, seven of which were in one-to-one correspondence with the seven in the other row. The S was asked to compare the number of blocks in the two rows. Then the row containing seven blocks was spread out so that it was as long as the row containing nine blocks, and the S was again asked to compare the number of blocks in the two rows. (See Figure 1)

C. Nonequivalence II: The S was shown two rows of blocks. One row contained seven blocks; and the other contained nine blocks, seven of which were in one-to-one correspondence with the seven in the other row. The S was asked to compare the number of blocks in the two rows. Then the row with seven blocks was spread out so that it was about twice as long as the row containing nine blocks, and the S was again asked to compare the number of blocks in the two rows. (See Figure 1)

Problem type	Before transformation	After transformation
Equivalence	....	....
Nonequivalence I	....	.. . .
Nonequivalence II	....	. . . .

Fig. 1. Conservation of Discontinuous Quantity

II. Conservation of Discontinuous Quantity with Counting

A. Equivalence: The same task as in Item IA above except that the S was asked to determine the number of blocks in each row before the transformation. E pointed to one row and asked the S to count it and tell him how many blocks there were in that row. After the S did this, he was asked how many blocks there were in the other row. He was not told to count the blocks in the second row, but he was not prevented from counting them. After the blocks were spread out and the S had judged the relation between the number of blocks in the two rows, he was again asked how many blocks there were in each row.

B. Nonequivalence II: The same task as in Item IC above except just as in Item IIA, the S was asked to determine the number of blocks in the two rows. The procedures were the same as in Item IIA and the row with seven blocks was always the first one counted.

III. Conservation of Continuous Quantity

A. Equivalence: The S was shown two identical glasses containing equal amounts of water and was asked to compare the amount of water in the two glasses. If he said that there was more water in one of the glasses, some water was poured from this glass into the other glass; and this process was repeated until the S agreed that there was the same amount of water in the two glasses. Then one of the glasses of water was poured into a taller, narrower glass, and the S was again asked to compare the amounts of water.

B. Nonequivalence I: The S was shown two identical glasses containing unequal amounts of water and was asked to compare the amounts of water in the two glasses. Then the glass containing the smaller amount of water was poured into a taller, narrower glass such that the height of the water was the same as the height of water in the glass containing more water, and the S was again asked to compare the two amounts of water.

C. Nonequivalence II: The S was shown two identical glasses containing unequal amounts of water and was asked to compare the amount of water in the two glasses. Then the glass containing the smaller amount of water was poured into a taller, narrower glass such that the height of the water was higher than the height in the glass containing more water, and the S was again asked to compare the two amounts of water.

IV. Measurement with Visibly Different Units

A. Equivalence: The S was shown two glasses containing equal amounts of water and was asked to compare the amounts of water in the two glasses. If he said that there was more water in one of the glasses, some water was poured from this glass into the other glass; and this process was repeated until the S agreed that there was the same amount of water in the two glasses. Then the water in each glass was measured into two opaque containers using visibly different units of measure so that one glass of water measured three units and the other measured five. Then the S was again asked to compare the two amounts of water.

B. Nonequivalence I: The S was shown two glasses containing unequal amounts of water and was asked to compare the amount of water in the two glasses. Then the water in each glass was measured into two opaque containers using visibly different units of measure such that both glasses measured three units. Then the S was again asked to compare the two amounts of water.

C. Nonequivalence II: The S was shown two glasses containing unequal amounts of water and was asked to compare the amount of water in the two glasses. Then the water in each glass was measured into two opaque containers using visibly different units of measure so that the greater quantity of water measured three units and the other measured four. Then the S was again asked to compare the two amounts of water.

V. Measurement with Indistinguishably Different Units

A. Equivalence: The same task as in Item IVA except the smaller unit appeared larger. One glass measured five units and the other measured four.

B. Nonequivalence I: The same task as in Item IVB except the smaller unit appeared larger. Both glasses measured four units.

C. Nonequivalence II: The same task as in Item IVC except the smaller unit appeared larger. The greater quantity of water measured six units and the other measured seven.

VI. Measurement of Unequal Appearing Quantities with the Same Unit

A. Equivalence: The S was asked to compare two equal quantities of water in two different shaped containers, one tall and narrow and the other short and wide (i.e. the final state in

Item IIIA above). Then the water in each glass was measured into two opaque containers using the same unit (each glass measured four units), and the S was again asked to compare the two amounts of water.

C. Nonequivalence II: The S was asked to compare two unequal amounts of water in the two different shaped containers (the final state in Item IIIC above). Then the water in each container was measured into two opaque containers using the same unit (the glass that appeared to have more water measured four units and the other measured five), and the S was again asked to compare the two amounts of water.

VII. Measurement with the Same Unit into Apparent Inequality

A. Equivalence: Using the same unit of measure, four units of water were measured into two different shaped containers; and the S was asked to compare the two quantities of water.

C. Nonequivalence II: Using the same unit of measure, five units of water were measured into a short, wide container and four units were measured into a tall, narrow container; and the S was asked to compare the two quantities of water.

Note: The tasks in Item VII were simply the tasks in Item VI with the stimuli appearing in a different order.

Design

The tasks were administered in two distinct studies each of which was divided into two parts. This accomplished three purposes. 1) The results of the first study could be used in planning the second study. 2) Tasks could be administered at appropriate grade levels. 3) The number of tasks administered to each S was kept at a reasonable number.

In Study I all the conservation tasks were administered to a group of kindergarteners. Sixty-five Ss in Part A received both sets of discrete object conservation problems with Equivalence and Nonequivalence II relations. Twenty-four in Part B received conservation of

Table 1

Studies in which Specific Items Appeared

Type of Problem		Relation		
		Equivalence	Non-Equivalence I	Non-Equivalence II
Conservation	Discrete Objects	IA, IB	IB	IA
	Discrete objects counted before transformation	IA		IA
	Liquid quantity	IB, IIA	IIA	IB, IIA
Measurement with different units	Distinguishably different units	IIA, IIB	IIA	IIA, IIB
	Indistinguishably different units	IIA, IIB	IIA	IIA, IIB
Measurement with same unit	Into different shaped containers	IIB		IIB
	From different shaped containers	IIB		IIB

continuous quantity problems with Equivalence and Nonequivalence II relations and discrete object conservation problems with Equivalence and Nonequivalence I relations.

In Study II the conservation of continuous quantity problems and all the measurement problems were administered to a group of 129 first and second graders. Sixty-one Ss in Part A received all conservation of continuous quantity problems and both sets of problems in which quantities were measured with two different units. All three sets of problems were administered with each of the three relations. Sixty-eight Ss in Part B received all the measurement problems with Equivalence and Nonequivalence II relations.

Study I

Part A

Part A of Study I was a 2 X 2 repeated measures design. The factors were type of problem (conservation of discrete objects with counting and without counting) and relation (Equivalence and Nonequivalence II). Main effect comparisons were designed to test the effect of supplementary numerical information in conservation problems and differences between Equivalence and Nonequivalence II in discrete object conservation problems, and the interaction was designed to test whether assigning numbers differentially reinforces equivalence and nonequivalence relations.

Part B

Part B of Study I was designed to test for differences 1) between

Equivalence and Nonequivalence II with conservation of continuous quantity problems and 2) between Equivalence and Nonequivalence I with conservation of discrete objects. This part of the study was exploratory in nature. Based on the results of previous studies and a pilot study, it was predicted that continuous quantity conservation problems would be too difficult for most kindergarteners. Second, based on the results of the Carpenter (in press) study and the fact that in the final state of Nonequivalence I problems the row containing more elements actually looks like it has more because the elements are closer together, it was predicted that the Nonequivalence I problems would be very easy and therefore differences between Equivalence and Nonequivalence I problems would be very pronounced. For these reasons relatively few Ss were assigned to this group.

Subjects

Study I was run between February 12 and March 5, 1971 in Stoughton, Wisconsin, a rural community near Madison, Wisconsin, with a population of about 6,000. The Ss for the study were selected from the four kindergarten classes in one of the eight elementary schools serving Stoughton and the surrounding areas. The sample, which included all kindergarten students in the school except six who were absent on the testing days, consisted of 89 Ss whose ages ranged from 5 yrs., 6 mos. to 6 yrs., 11 mos., with mean age 6 yrs., 1 mos.

Ss were randomly assigned to two groups, 65 Ss to Part A and 24 to Part B. Each S within each group received the same basic set of problems; however, the order of the problems and the protocols, procedures, and materials used were systematically varied; and Ss within each group were randomly assigned to one of the variations.

Part A Procedures

One difficulty in administering a series of problems to the same S is that there may be some sort of interaction between the problems, and problems given earlier may affect the results of subsequent problems. The results of a pilot study also indicated that Ss may respond to all subsequent items on the same basis as their original response.

In order to reduce any possible order effect, problems in Part A were administered in four different sequences. (See Table 2) Each of the four problems in Part A of Study I was the first problem in one of the sequences. The second problem in each sequence was the same problem type (discrete object conservation counting or not counting) with the opposite relation. The third problem in each sequence was identical to the first. This provided a test of the reliability of each item. The fourth problem in each sequence was the opposite problem type as the first three problems with the same relation as the second problem in the sequence. The fifth item in each sequence was the same problem type as the fourth but the opposite relation. Thus, all Ss received the same set of original problems. In each sequence the first problem administered was repeated; therefore, each item in the

Table 2

Sequences of Items in Study IA

	Order of Item	Sequence Groups			
		1	2	3	4
Day 1	1	Equivalence not counted	Nonequiv. II not counted	Equivalence counted	Nonequiv. II counted
Day 2	2	Nonequiv. II not counted	Equivalence not counted	Nonequiv. II counted	Equivalence counted
	3	Equivalence not counted	Nonequiv. II not counted	Equivalence counted	Nonequiv. II counted
	4	Nonequiv. II counted	Equivalence counted	Nonequiv. II not counted	Equivalence not counted
	5	Equivalence counted	Nonequiv. II counted	Equivalence not counted	Nonequiv. II not counted

study was repeated in one of the sequences. In each sequence the odd problems employed one relation and the even the other relation. The first three problems in the sequence were of one problem type and the last two the other type. In this way, *S*s starting with the noncounting problems completed them before counting was introduced, as it was predicted that once counting was introduced it would affect all subsequent problems.

The first problem in each sequence was administered to all Ss on February 15, 1971. The next four problems were administered over two weeks later on March 3, 4, and 5. It was assumed that this break in time would minimize any interaction between items and thereby provide a more sensitive test of the differences between the relations used in the first and second problems.

All items were administered in a small room apart from the classroom by one E, who was a stranger to the Ss. The S sat at a table opposite the E. Procedures and protocols were kept as consistent as possible between items; however, certain procedures were systematically varied between Ss in order to control for responses based on experimental variables.

Piaget (1968) and Siegel & Goldstein (1969) found that young children tend to respond to the last choice available to them in a conservation problem. Thus, if the E says "Are there the same number of blocks in the two rows or does one have more?", the S responds that one has more, because "more" was the last choice given to him.

Therefore, half of the Ss in each sequence were asked, "Are there the same number of blocks in each row or does one row have more?" and the other half asked, "Does one row have more blocks in it or are there the same number of blocks in each row?". For each S the "same-more" order was the same for all problems.

Similarly, the positions of the blocks in rows was systematically altered between Ss. The blocks were always in parallel rows between the E and the S; however, for half of the Ss in each of the eight groups (four sequences X two more-same protocol variations), the row toward the S was always the one to be transformed; and for the other half the row toward the E was always the one to be moved.

A third procedural variation between Ss was in the materials used. The materials were small colored blocks. In any given problem the blocks were all the same color; however, there were three colors--red, blue, and brown; and different colored blocks were used for different problems. The colors of the blocks appeared in two orders in each of the 16 groups (four sequences X two more-same protocol variations X two row positions). One order was: red, blue, brown, red, blue; and the other was: blue, red, brown, blue, red. It was not predicted that color would have any effect on responses, but it was very easy to control.

All other procedures were kept constant between Ss and between items. The blocks were arranged before the S; he made his original judgement; the blocks were transformed; and after he again judged the relative number of blocks, he was asked, "Why do you think they are the same?" or "Why do you think there are more there?"; and his response was recorded. For statistical purposes responses were scored as correct if the S chose the correct relation, even though he was not able to verbalize his reason.

There was one exception to this procedure. If the S chose the shorter row as having more blocks in both Equivalence and Nonequivalence II problems, he was correct in the Nonequivalence II problem and wrong in the Equivalence problem. Therefore, *Ss* who always chose the shorter row as more were given a sixth item which was identical to the Nonequivalence II item except that the more numerous row was spread out. For these *Ss* their Nonequivalence II responses were not scored as correct unless they also correctly answered this sixth question.

Part B Procedures

Procedures for Part B of Study I were similar to those employed in Part A. The same *E* administered the items under the same conditions as in Part A. Basically the same protocols and variations were used. The order of "more" and "same" was varied between *Ss*. In the discrete object problems the positions of the rows and the colors of the blocks were systematically varied between *Ss*. The same criteria of evaluating correct responses without regard to explanations was employed.

The same blocks used in Part A were used in the discrete object problems. Clear plastic glasses, whose capacity was about a cup, were used in the continuous quantity problems. They were filled about two-thirds full in the Equivalence problem, and in the Nonequivalence II problem the less full glass was also about two-thirds full and the other glass was filled about three-fourths of an inch higher. The tall, narrow container that one of the quantities was poured into had a diameter about one-half that of the glasses and was about three times as tall. Thus, when the water was poured into this container, its level was raised by about a factor of four.

Each *S* responded to four problems, one on February 15, 1971 and the other three over two weeks later on March 3, 4, or 5. As in Part A, items appeared in four orders. In all cases the first and third items were continuous quantity conservation items, with half the *S*s receiving Equivalence first and the other half receiving Nonequivalence II first. In each of these groups half the *S*s received the discrete object Equivalence problem as the second item and the Nonequivalence I item as the fourth and the other half received the Nonequivalence I item as the second item and the Equivalence item as the fourth.

Table 3

Sequences of Items in Study IB

	Order of Items	Sequence Groups			
		1	2	3	4
Day 1	1	Continuous quantity Equivalence	Continuous quantity Nonequiv. II	Continuous quantity Equivalence	Continuous quantity Nonequiv. II
Day 2	2	Discrete objects Equivalence	Discrete objects Equivalence	Discrete objects Nonequiv. I	Discrete objects Nonequiv. I
	3	Continuous quantity Nonequiv. II	Continuous quantity Equivalence	Continuous quantity Nonequiv. II	Continuous quantity Equivalence
	4	Discrete objects Nonequiv. I	Discrete objects Nonequiv. I	Discrete objects Equivalence	Discrete objects Equivalence

Study II

Part A

Part A of Study II was a 3 X 3 repeated measures design. The factors were problem type (continuous quantity conservation and the two measurement problems with different units) and relation (Equivalence and Nonequivalence I and II).

Based on past research and the results of Study I (reported below) it was predicted that:

1) There are no significant differences between performance on Equivalence and Nonequivalence II problems.

2) Nonequivalence I problems are easier than both Equivalence and Nonequivalence II problems in all cases except the case in which different quantities are measured with different units, and the correct relation between the units is not visibly distinguishable.

3) Problems in which quantities are measured with different units are easier if the correct relation between the units is visually obvious than if the correct relation is not obvious.

4) It is not clear what the relation between continuous quantity conservation and measurement problems is.

Based upon these predictions the model represented in Table 4 was hypothesized.

The hypotheses tested in terms of this model with error components added (Table 5) are:

1) $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 0$. There are no significant differences between Equivalence and Nonequivalence II problems or between any of the three relations in which the larger unit is not distinguishable.

2) $\Theta = 0$. (Null) There is no significant difference between measurement problems in which it is possible to visually

Table 4

Hypothesized Item Means for Items in Study IIA

Problem type	Relation		
	Equivalence	Nonequiv. I	Nonequiv. II
Conservation continuous quantity	$\mu+\theta+\alpha$	$\mu+\theta+\alpha+\gamma+\beta$	$\mu+\theta+\alpha$
Measurement with distinguishably different units	$\mu+\theta$	$\mu+\theta+\gamma$	$\mu+\theta$
Measurement with indistinguishably different units	μ	μ	μ

distinguish the correct relation between different units and those in which it is not.

3) $\alpha = 0$. There is no significant difference between conservation and corresponding measurement problems.

4) $\gamma = 0$. (Null) There is no significant difference between problems involving Nonequivalence I relations and corresponding problems involving Equivalence or Nonequivalence II relations.

5) $\beta = 0$. Any differences between conservation and measurement problems for Nonequivalence I are of the same magnitude as those that exist for other relations.

Table 5

Hypothesis Test Model for Item Means in Study IIA

Problem type	Relation		
	Equivalence	Nonequiv. I	Nonequiv. II
Conservation continuous quantity	$\mu+\Theta+\alpha-\Delta_4$	$\mu+\Theta+\alpha+\gamma+\beta$	$\mu+\Theta+\alpha+\Delta_4$
Measurement with distinguishably different units	$\mu+\Theta-\Delta_3$	$\mu+\Theta+\gamma$	$\mu+\Theta+\Delta_3$
Measurement with indistinguishably different units	$\mu+\Delta_1+\Delta_2$	$\mu-\Delta_1+\Delta_2$	$\mu-2\Delta_2$

Part B

Part B of Study II was a 2 X 4 repeated measures design. The factors were problem type (all four measurement problems) and relation (Equivalence and Nonequivalence II).

The following predictions were added to those already noted above in Part A:

1) Problems in which quantities are measured with the same unit are easier than those in which two different units are used.

2) Problems in which the correct cues appear last are easier than

problems in which incorrect cues appear after the correct ones.

Based on these predictions and those in Part A, the model represented in Table 6 was hypothesized.

The hypotheses tested in terms of this model with error components added (Table 7) are:

1) $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 0$. There is no significant difference between Equivalence and Nonequivalence II problems.

2) $\Theta = 0$. (Null) There is no significant difference between measurement problems in which it is possible to visually

Table 6

Hypothesized Item Means for Items in Study IIB

Problem type	Relation	
	Equivalence	Nonequiv. II
Measurement with distinguishably different units	$\mu + \Theta$	$\mu + \Theta$
Measurement with indistinguishably different units	μ	μ
Measurement into different shaped containers	$\mu + \Theta + \gamma$	$\mu + \Theta + \gamma$
Measurement from different shaped containers	$\mu + \Theta + \gamma + \alpha$	$\mu + \Theta + \gamma + \alpha$

distinguish the correct relation between different units and those in which it is not.

3) $\gamma = 0$. (Null) There is no significant difference between measurement problems in which the correct cues are numerical and those in which the correct cues are visual.

4) $\alpha = 0$. (Null) There is no significant difference between measurement problems in which the correct measurement cues appear before distracting visual cues and those in which they appear after the distracting visual cues.

Table 7

Hypothesis Test Model for Item Means in Study IIB

Problem type	Relation	
	Equivalence	Nonequiv. II
Measurement with distinguishably different units	$\mu+\Theta-\Delta_2$	$\mu+\Theta+\Delta_2$
Measurement with indistinguishably different units	$\mu-\Delta_1$	$\mu+\Delta_1$
Measurement into different shaped containers	$\mu+\Theta+\gamma-\Delta_3$	$\mu+\Theta+\gamma+\Delta_3$
Measurement from different shaped containers	$\mu+\Theta+\gamma+\alpha-\Delta_4$	$\mu+\Theta+\gamma+\alpha+\Delta_4$

Subjects

Study II was run over a nine day period between April 20 and April 30, 1971 in Edgerton, Wisconsin, a predominantly rural community about 20 miles from Madison, Wisconsin, with a population of about 4,000. The Ss for the study were selected from three of the five first grade classes and two of the five second grade classes in one of the two elementary schools serving the community. The sample, which included all students in the five classes except three who were absent on the testing days, consisted of 75 first graders and 54 second graders. The age range of the first graders was 6 yrs., 5 mos. to 9 yrs., 8 mos. with mean age 7 yrs., 5 mos., and the range of the second graders was 7 yrs., 7 mos. to 9 yrs., 5 mos. with mean age 8 yrs., 4 mos.

Procedures

Ss were randomly assigned to two groups, 61 Ss to Part A and 68 to Part B. Each S within each group received the same basic set of problems; however, the order of the problems was randomized for each S. In addition Ss were randomly assigned protocol and procedural variations. There was the same "more-same" variation described in Study I, and for some Ss the smaller quantity was always measured first in nonequivalence problems, and for others the larger quantity was always measured first. Both of these variations were randomly assigned to Ss.

Problems were administered in two sittings. Ss in Part A received five problems the first day and four several days later, and Ss in

Part B had four problems the first day and four the second. As in Study I answers were judged correct or incorrect without regard to the explanations given.

The materials used in the conservation problems were the same as those used in Study I. In the measurement problems the same two glasses were used as were used in the conservation problem. The distinguishably different unit was a small plastic glass that held about an ounce and a half of water. The bigger unit was another plastic glass about the same diameter but taller that held one and two-thirds as much water. The smaller plastic glass also served as the larger of the indistinguishably different units. The other was a glass shot glass, which appeared quite a bit larger, but actually only held about four-fifths as much water. One of the two standard glasses and the tall, narrow glass from the conservation problem and the smaller plastic unit were used in the problems in which quantities were measured with the same unit.

Analysis

Item totals, reasons for responses, and types of errors were recorded for each item. The following categories were used to classify reasons for correct responses:

1) Reversibility: If the quantities were transformed back to their former state, they would again appear in the correct relation (equal or unequal).

2) Statement of operation performed: The blocks were just spread out or the water was just poured into a different container and this did not change the relation between the quantities.

3) Addition--subtraction: Nothing was added or taken away.

4) Compensation, proportionality: The row of blocks was longer but the blocks were farther apart. The liquid was higher but the container was narrower. One measured more units but the units were smaller.

5) Sameness of quantity: They're the same blocks. It's the same water. Also, there are still seven in both rows.

6) Reference to previous state: They were the same before when the blocks were in one-to-one correspondence or the water was in identical glasses.

7) No reason, unclassifiable: No reason was given or an incomprehensible reason was given.

Incorrect responses were sorted into two broad categories:

1) Dominant dimension: Ss incorrectly chose a)the longer row of blocks b)the taller container of water c)the quantity that measured the greater number of units.

2) Secondary dimension: Ss incorrectly chose a)the more dense row of blocks b)the wider container c)the quantity measured with the larger unit.

Confidence intervals were plotted for individual items and for the contrasts tested in the analysis. This allows one to determine the probable per cent of correct responses for items and probable magnitude of the difference in performance between items for the population.

The hypotheses were tested using a multivariate analysis of variance program due to Finn (1967). Although the nominal data of this study do not fit all the assumptions required for parametric statistics, Cochran (1950) has found that parametric statistics can generally be applied to nominal data if one is cautious about attaching importance to marginally significant results. Since for several of the contrasts it was most interesting to show that significant differences did not exist, nonparametric statistics were undesirable in that failure to find significance could have resulted from lack of power of the statistical test.

In addition to the fundamental test for significance, each of the hypotheses was tested to determine whether it was significantly influenced by sex, grade, or order of the items. If any of these variables were found to be significant, the hypothesis was retested with the levels of the factors being balanced with respect to the significant variables.

For both parts of Study I, items appeared in only four orders; consequently, there was no difficulty in identifying and testing for order effects. Since the order of items in Study II was randomized, every S received a different order of items, thereby eliminating the feasibility of partitioning into each distinct order. Therefore, the order groups for Study II were determined by a procedure proposed by Zimiles (1966). He found that the first item administered often significantly influenced performance on all subsequent items. Ss administered easier first items performed better on all subsequent items than Ss administered a more difficult first item. No differences were found, however, due to variations in the second item administered. Thus, in Study II Ss were partitioned into order groups based on the first item they received. Note that this procedure also identifies the four order groups in Study I.

Chapter IV

RESULTS

Study IA

The results of individual items in Study IA, the reasons given for responses, and the types of errors are summarized in Table 8.

On the counting Equivalence problem, 44 of the 65 Ss counted both rows of blocks even though the rows were arranged in one-to-one correspondence and the Ss were not specifically asked to count the second row. Only 2 Ss were able to determine that there were nine blocks in the more numerous row in the Nonequivalence II problem without counting the entire row. About half the Ss (33 in the Equivalence problem and 37 in the Nonequivalence II problem) recounted at least one of the rows in order to determine the number of blocks in the two rows after the transformation.

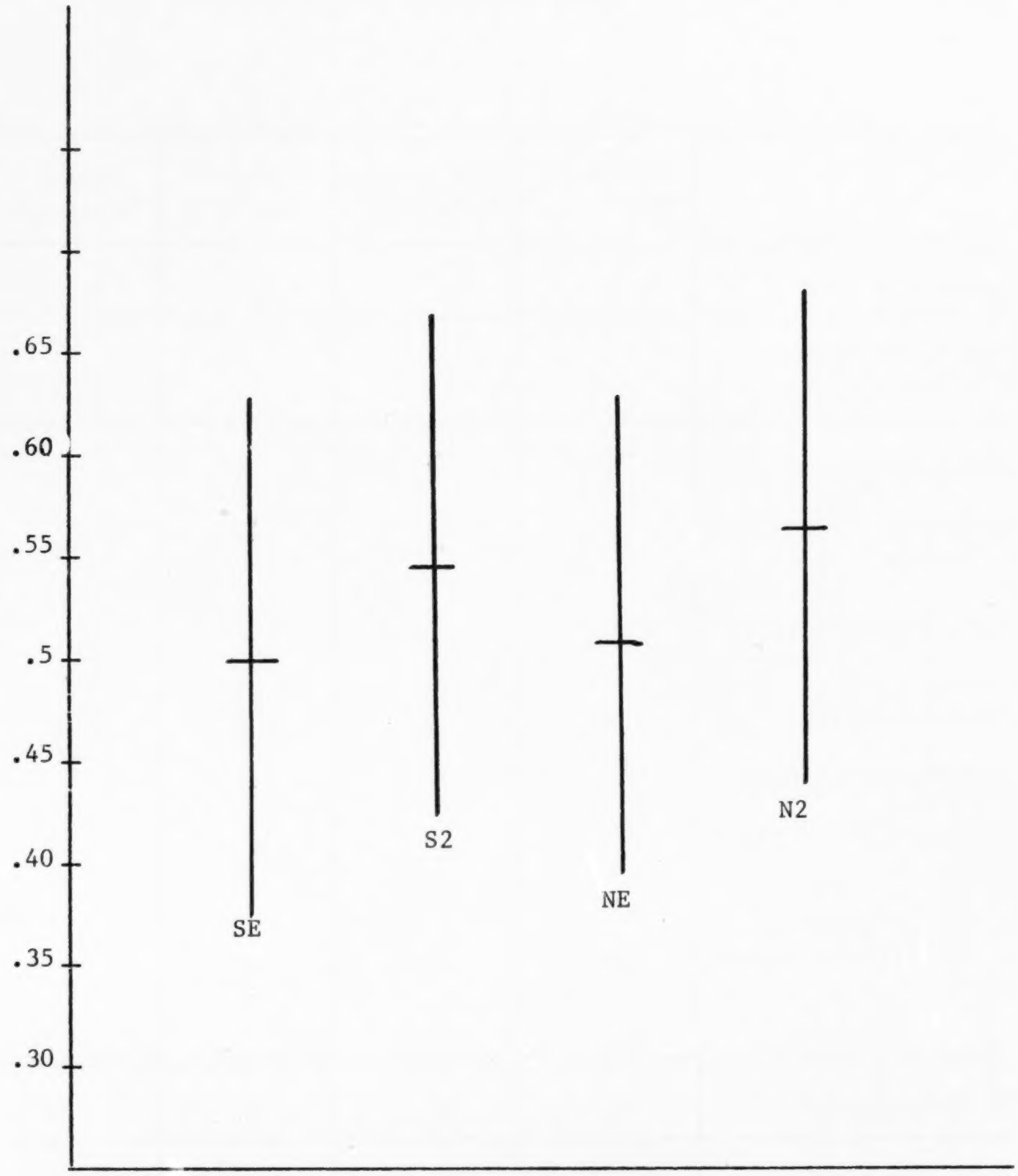
The means for individual items surrounded by 95% confidence intervals have been plotted in Figure 2. Since individual items are scored on a 0,1 basis, the mean can be interpreted as representing the fraction of Ss correctly responding to the item. Similarly the confidence intervals can be interpreted in terms of per cents. For example, there is a 95% probability that between 37% and 56% of the population would respond correctly to Item 1 of Study IA.

Table 8 and Figure 2 indicate that mean performance on each of the four items was similar. Furthermore, Ss were extremely consistent in their responses to the set of items. Eighty-five per cent of the Ss

Table 8

Number of Subjects in the Major Response Categories in Study IA

Item	Not count Equiv.	Not count Nonequiv.	Count Equiv.	Count Nonequiv.
Total correct	31	35	35	37
Reason for correct response				
Reversibility	0	0	0	0
Statement of operation performed	7	6	9	8
Addition-subtraction	5	6	8	8
Compensation, proportionality	8	8	6	9
Sameness of quantity	0	1	1	1
Reference to previous state	7	7	6	5
No reason given or unclassifiable reason given	4	7	5	6
Total incorrect	34	30	30	28
Type of error				
Longer row	28	26	26	26
More dense row	6	4	4	2



S = Conservation of discrete objects without counting
N = Conservation of discrete objects with counting
E = Equivalence
2 = Nonequivalence II

Fig. 2. Confidence Intervals for Items in Study IA

tested gave the same response to all four items with 30 Ss correctly answering all four and 25 Ss missing all four. Three Ss missed both non-counting items but responded correctly to both the items in which the blocks were counted.

Ss were even more consistent in their responses to problems of the same type but different relations. In both the counting and non-counting situations, 94% of the Ss gave the same response to the Equivalence and Nonequivalence II problems. This was even slightly better than performance on the repeated items. Ninety-one per cent of the Ss gave the same response on both administrations of the item that was repeated for them, three Ss missed the item the first time it was administered and answered it correctly on the second administration, and three Ss answered correctly the first but were wrong the second.

Analysis of variance summarized in Table 9 indicates there exists

Table 9

MANOVA--Relations, Problem Type, and Interaction in Study IA

Source	df	MS	F	p<
Multivariate	3,55		1.9896	.1263
Relation	1	.0036	.2697	.6056
Problem Type	1	.0346	4.6985	.0344
Interaction	1	.0036	.5221	.4730

Degrees of freedom for error = 57

a significant difference between counting and non-counting problems[1] but no significant difference between the Equivalence and Nonequivalence II relations and no significant interaction between counting and the type of relation.

The 95% confidence interval for the difference between counting and non-counting problems plotted in Figure 3 reveals that, although significant, the difference between performance on counting and non-counting problems is extremely small, occurring in less than 5% of the population.

Similarly the confidence interval for the difference between Equivalence and Nonequivalence II reveals that not only is the hypothesis of no difference between Equivalence and Nonequivalence II problems not rejected, but the hypothesis that the difference occurs in less than 4% of the population can be accepted with 95% confidence.

The analysis summarized in Table 10 indicates that the results summarized in Table 9 were consistent across sex and order of items.

Zimiles' (1966) procedure of looking for significant differences in conservation problems by considering differences in performance between groups receiving different first items is summarized in Table 11 along with contrasts between sexes. This analysis indicates that there are no significant differences in mean performance on the set of items between boys and girls or between Ss receiving the problems in different orders.

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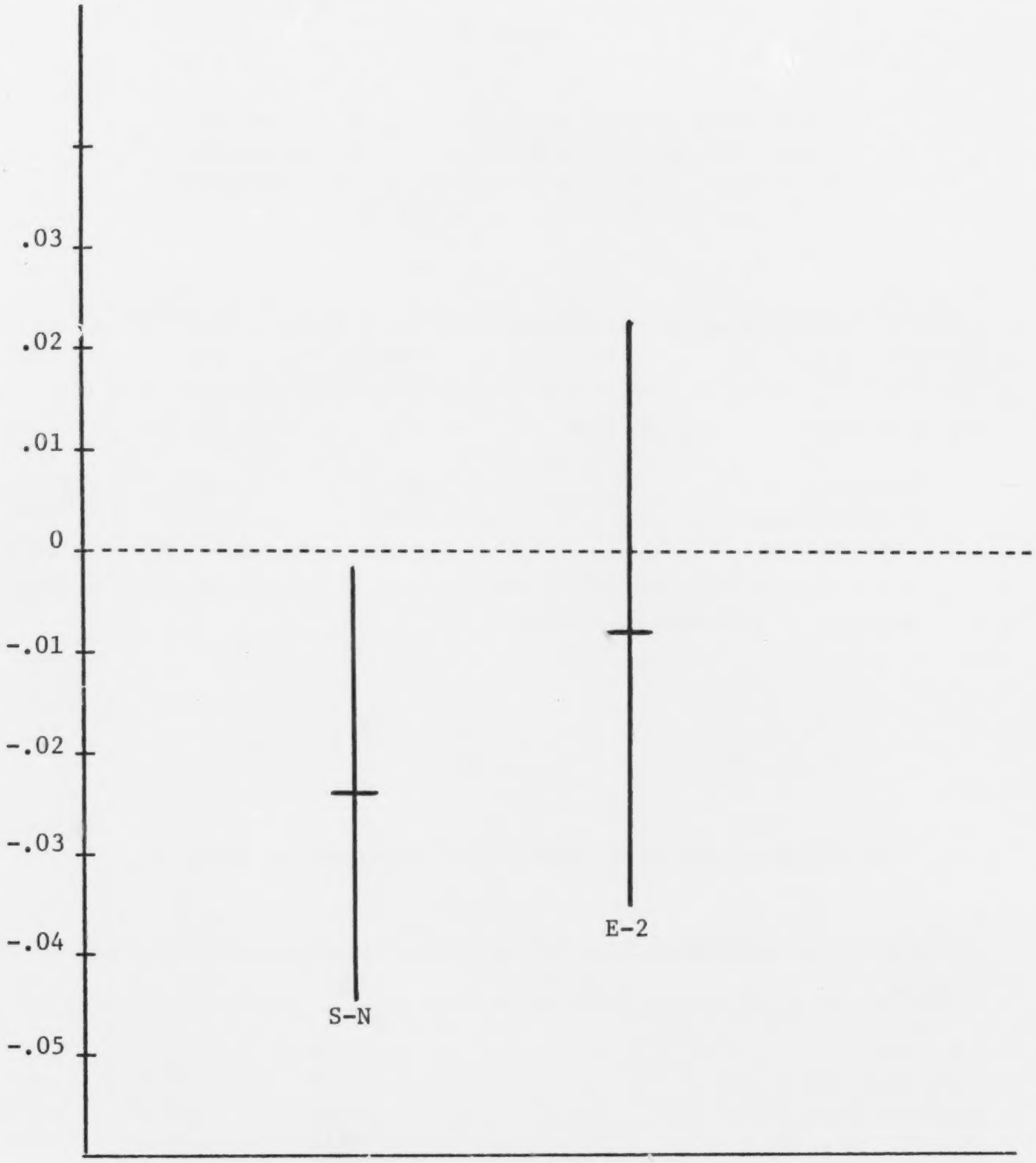
[1] Throughout this investigation "significant difference between problems A and B" should be interpreted as "significant difference between the mean number of correct responses to problems A and B."



S = Conservation of discrete objects without counting
N = Conservation of discrete objects with counting
E = Equivalence
2 = Nonequivalence II

Fig. 3. Confidence Intervals for Hypotheses in Study IA

Table 10

MANOVA--Combined Analysis of Sex, Order of Items, and Sex-Order Interaction for Relation, Problem Type, and Relation-Problem Type Interaction Contrasts in Study IA

Source	df	MS	F	p<
Multivariate	21,158.4804		1.2917	.1180
Relation	7	.0065	.8812	.5272
Problem Type	7	.0262	1.8369	.0977
Interaction	7	.0109	1.4778	.1937

Degrees of freedom for error = 57

Table 11

ANOVA--Mean Score of Four Items Combined in Study IA

Source	df	MS	F	p<
Sex (A)	1	.7028	3.2374	.0773
Relation Order (B)	1	.7559	3.4819	.0672
Problem Type Order (C)	1	.0140	.0643	.8008
AB, AC, BC, ABC	4	.0853	.3927	.8131
Within Cells	57	.2171		

A third analysis contrasts performance on the first item administered between groups of Ss receiving different first items. This analysis, which has the advantage of completely eliminating any possible interaction between items is summarized in Table 12 and also indicates no significant differences between relations or between problem types.

Table 12

ANOVA--First Item Received in Study IA

Source	df	MS	F
Relation	1	.855	3.35
Problem Type	1	.171	1<
Interaction	1	.171	1<
Within Cell	66	.254	

$F_{.95;1,66} = 3.99$

Study IB

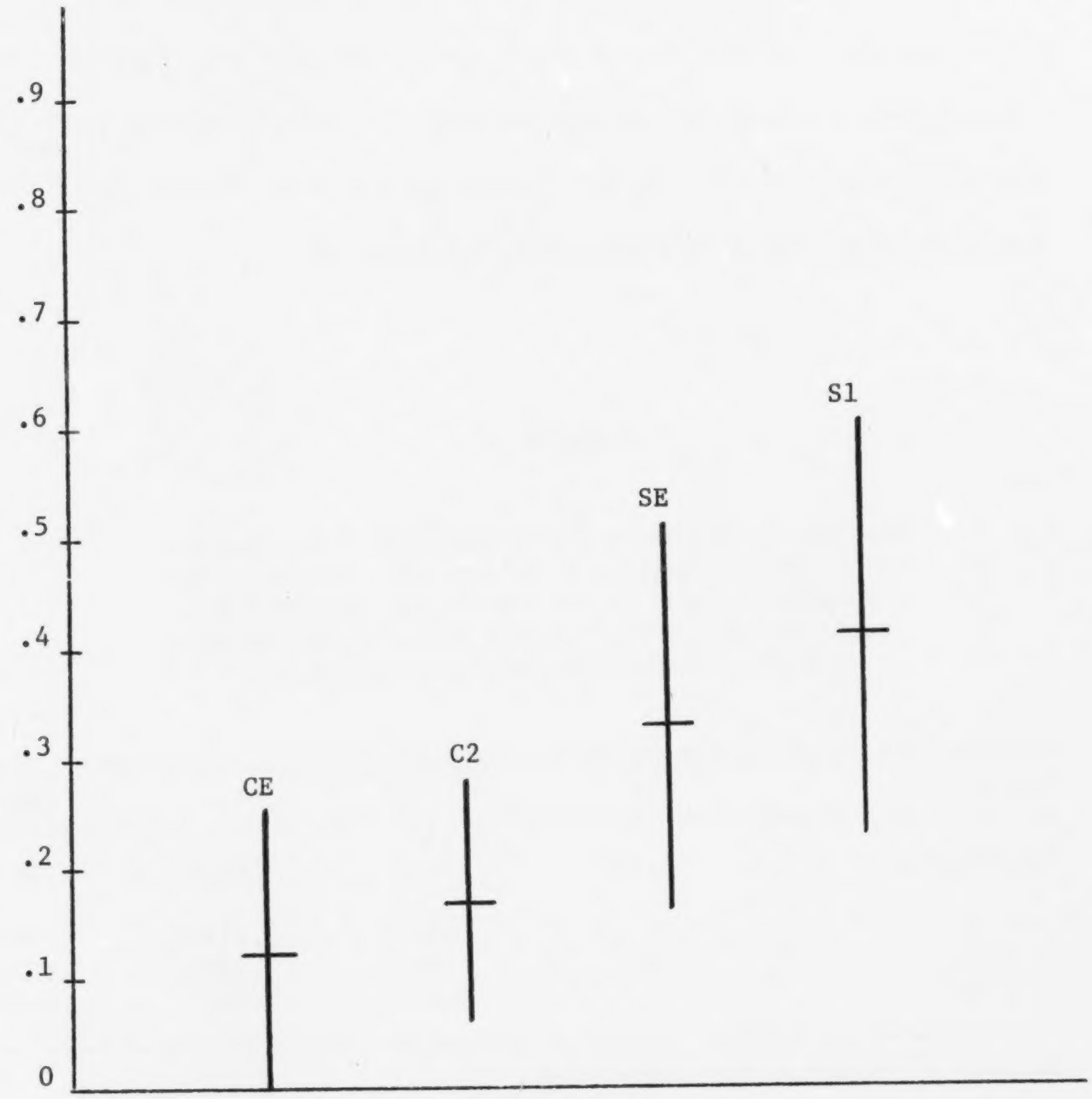
The results of individual items in Study IB, the reasons given for responses, and the types of errors are summarized in Table 13; and the means for individual items surrounded by 90% confidence intervals have been plotted in Figure 4.

Only one S correctly answered all four items while 12 Ss missed all four. Four Ss missed both continuous quantity items but correctly

Table 13

Number of Subjects in the Major Response Categories in Study IB

Item	Continuous Equiv.	Continuous Nonequiv.II	Discrete Equiv.	Discrete Nonequiv.I
Total correct	4	4	8	10
Reason for correct response				
Reversibility	0	0	0	0
Statement of operation performed	0	0	2	3
Addition-subtraction	0	0	3	2
Compensation, proportionality	0	0	1	1
Sameness of quantity	0	0	0	0
Reference to previous state	1	2	2	1
No reason given or unclassifiable reason given	3	2	0	3
Total incorrect	20	20	16	14
Type of error				
Length of row or height of water	20	20	12	13
More dense row or wider container	0	0	4	1



S = Conservation of discrete objects
C = Conservation of continuous quantity
E = Equivalence
1 = Nonequivalence I
2 = Nonequivalence II

Fig. 4. Confidence Intervals for Items in Study IB

answered both discrete object items, and 2 Ss only answered the discrete object Nonequivalence II item correctly.

The analysis summarized in Table 14 indicates there is a significant difference between continuous quantity Equivalence and discrete object Equivalence problems but no significant difference between Equivalence and Nonequivalence II with continuous quantity or between Equivalence and Nonequivalence I with discrete objects.

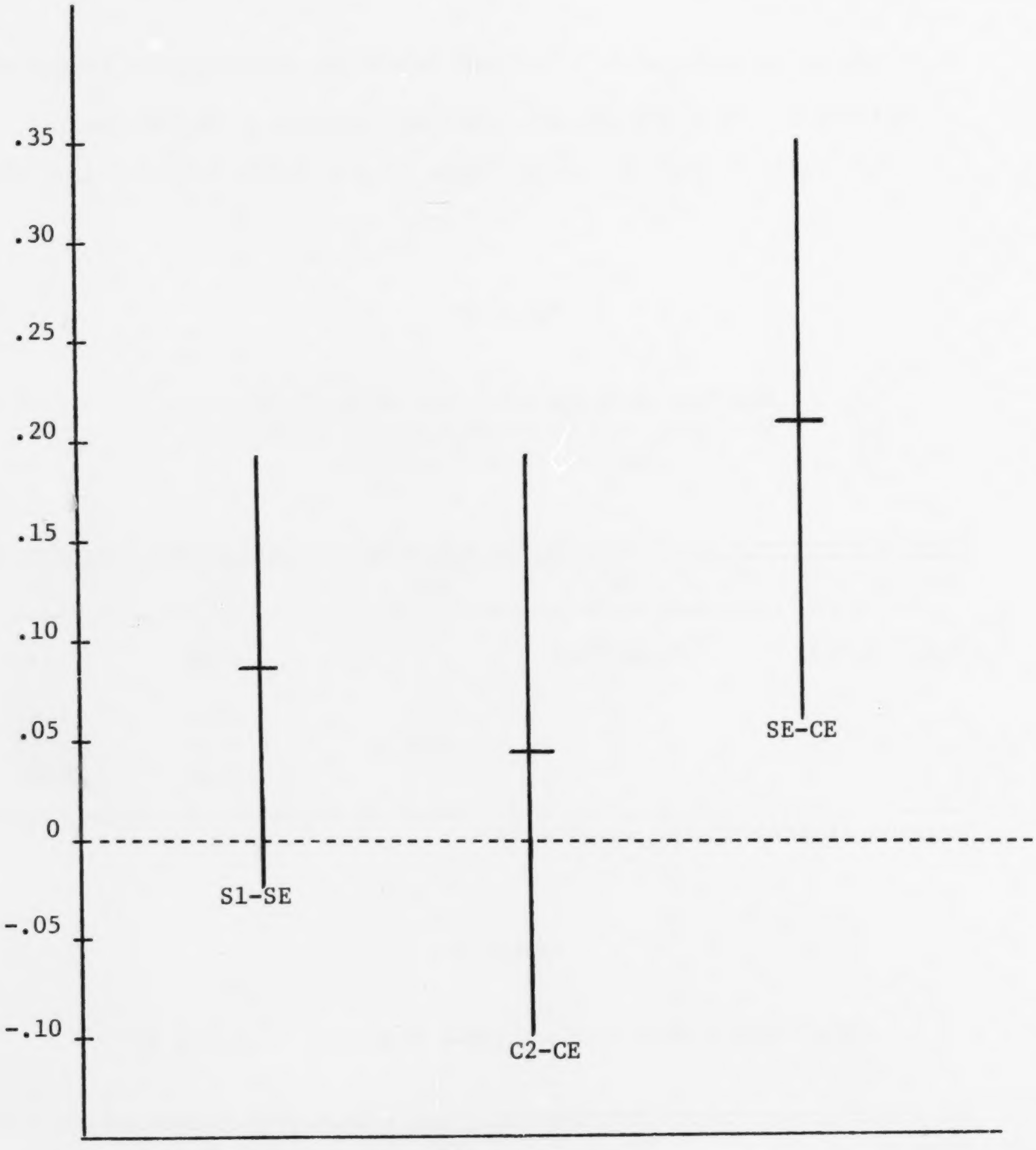
Table 14

MANOVA--Equivalence-Nonequivalence I Contrasts for Discrete Objects Problems (A), Equivalence-Nonequivalence II Contrasts for Continuous Quantity (B), and Discrete Object-Continuous Quantity Contrasts for Equivalence (C)

Source	df	MS	F	p<
Multivariate	3,14		2.6539	.0892
A	1	.1667	1.8187	.1964
B	1	.0417	.2381	.6323
C	1	1.0417	5.9524	.0268

Degrees of freedom for error = 16

The confidence intervals in Figure 5 indicate, however, that there is a 10% chance that differences between relations may occur in as much as 19% of the population. Thus, although the hypotheses



S = Conservation of discrete objects
C = Conservation of continuous quantity
E = Equivalence
1 = Nonequivalence I

Fig. 5. Confidence Intervals for Hypotheses in Study IB

of no differences between relations cannot be rejected, it should not be concluded from these results that differences do not exist.

The analysis summarized in Table 15 and Table 16 indicates that

Table 15

MANOVA--Combined Analysis of Sex, Order of Items, and Sex-Order Interaction for Hypotheses in Table 14

Source	df	MS	F	p<
Multivariate	21,40.7506		.9799	.5049
A	7	.0524	.5714	.7688
B	7	.3083	1.7619	.1647
C	7	.1655	.9456	.5002

Degrees of freedom for error = 16

Table 16

ANOVA--Mean Score of Four Items Combined in Study IB

Source	df	MS	F	p<
Sex (A)	1	.0234	.1915	.6676
Order (B)	3	.1050	.8582	.4828
AB	3	.0043	.0354	.9908
Within Cells	16	.1224		

the results in Table 14 are consistent across sex and order of items, and that there are no significant differences in mean performance between boys and girls or between Ss receiving problems in different orders.

Study IIA

The results of individual items in Study IIA, the reasons given for responses, and the types of errors are summarized in Table 17; and the means for individual items surrounded by 95% confidence intervals have been plotted in Figure 6.

There was much less diversity in the reasons for correct responses than there was in Study IA. Practically all the Ss either referred to the previous state of the quantities or noted the compensating relationship between unit size and the number of units or between height and width. Comparisons between reasons given by Ss who were successful on the items in which it was possible to distinguish the compensating relationship between unit size and number of units but were unsuccessful on problems in which it was not are enlightening. Seven of the eight Ss who either 1)correctly answered at least two of the measurement problems in which the larger unit was distinguishable but none of the problems in which it was not or 2)correctly answered all three of the problems in which the larger unit was distinguishable and at most one of the problems in which it was not gave compensation as the reason for at least one of their correct responses.

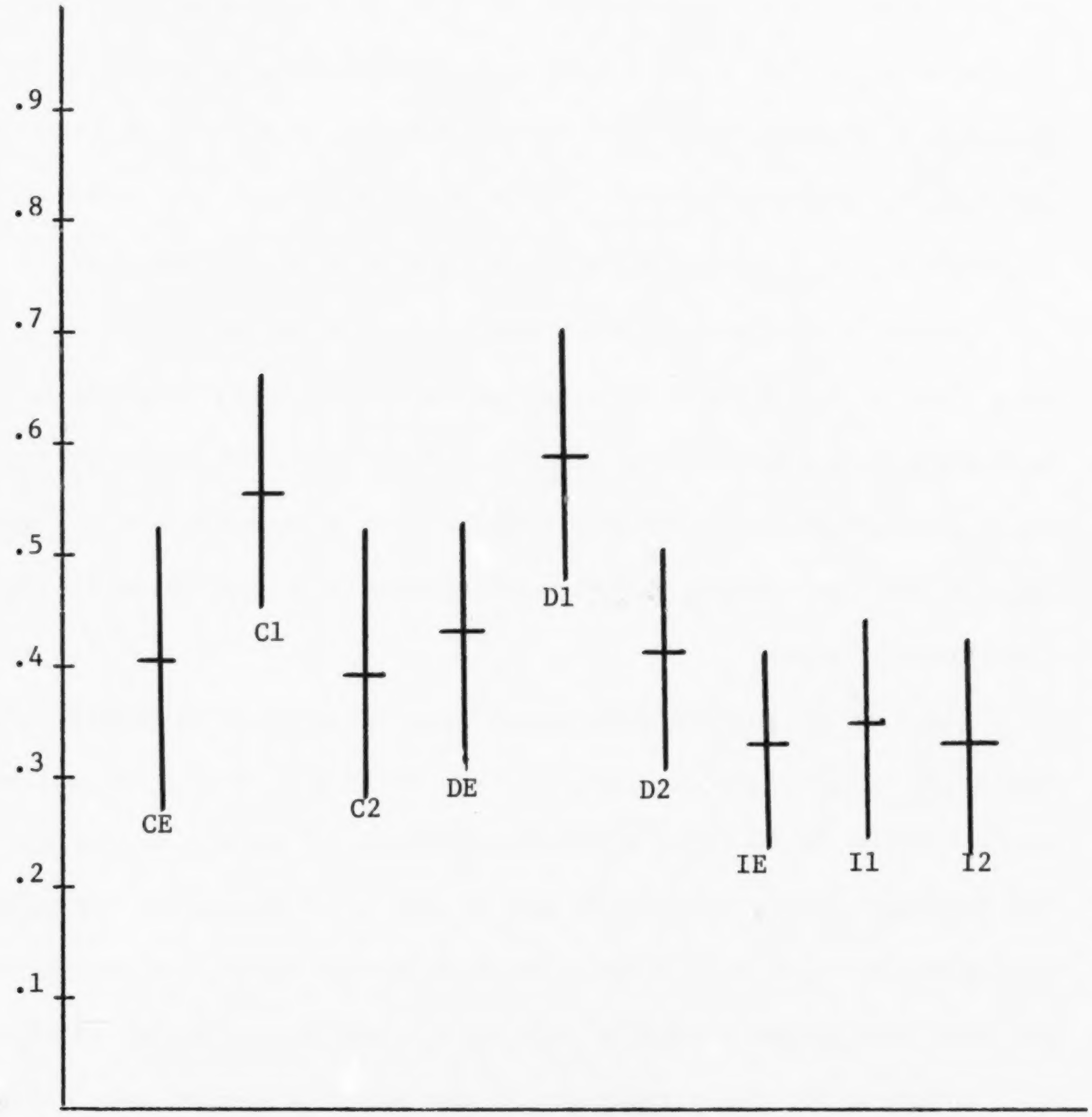
Between 79% and 87% of the Ss gave the same response to correspond-

Table 17

Number of Subjects in the Major Response Categories in Study IIA

Item	CE	C1	C2	DE	D1	D2	IE	I1	I2
Total correct	25	34	24	26	36	25	20	21	20
Reason for correct response									
Reversibility	0	0	0	0	0	0	0	0	0
Statement of operation performed	1	0	0	0	0	0	0	0	0
Addition-subtraction	0	0	0	0	0	0	0	0	0
Compensation, proportionality	10	13	5	9	10	8	0	0	1
Sameness of quantity	0	0	0	1	0	0	0	0	0
Reference to previous state	14	16	18	14	20	16	18	17	17
No reason given or unclassifiable reason given	0	5	1	2	6	1	2	4	2
Total incorrect	36	27	37	35	25	36	41	40	41
Type of error									
Taller container or greater number of units	36	25	35	33	23	32	40	36	39
Wider container or larger unit	0	2	2	2	2	4	1	4	2

C = Conservation of continuous quantity
D = Measurement with visibly different units
I = Measurement with indistinguishably different units
E = Equivalence
1 = Nonequivalence I
2 = Nonequivalence II



C = Conservation of continuous quantity
D = Measurement with visibly different units
I = Measurement with indistinguishably different units
E = Equivalence
1 = Nonequivalence I
2 = Nonequivalence II

Fig. 6. Confidence Intervals for Items in Study IIA

ing Equivalence and Nonequivalence II problems; and in the problem in which the larger unit was not distinguishable, 77% gave the same response to all three items. Between 70% and 79% of the Ss gave the same response to the conservation problems and the corresponding problems in which quantities were measured with distinguishably different units. One S who had difficulty counting ignored the measurement cues and correctly answered the measurement problems but was unable to conserve.

On the measurement problem with indistinguishably different units, only five of the Ss were able to use the information from the measurement operation to correctly identify the larger unit. The rest were unable to apply the inverse relationship between unit size and number of units to this problem and simply responded incorrectly on the basis of the unit that looked larger.

The test of the proposed model (See Table 5) is summarized in Table 18. These results indicate that there are no significant differences between Equivalence and Nonequivalence II relations for any of the problems tested or between any of the three relations for measurement problems with indistinguishably different units. Ninety-five per cent confidence intervals for the Δs have been plotted in Figure 7.

Analysis for the parameters of the model is summarized in Table 19. These results indicate that there are significant differences between Nonequivalence I and the other two relations for the conservation problem and for the measurement problems with distinguishably different units, but no significant differences were found between either of the

Table 18

MANOVA--Lack of Fit of Model for Study IIA

Source	df	MS	F	p<
Multivariate	4,24		.0769	.9887
Δ1	1	.0164	.0984	.7563
Δ2	1	.0164	.0450	.8336
Δ3	1	.0164	.1021	.7518
Δ4	1	.0164	.0830	.7755

Degrees of freedom for error = 27

Table 19

MANOVA--Parameters of Model for Study IIA

Source	df	MS	F	p<
Multivariate	5,23		30.5264	.0001
μ	1	6.7776	67.1332	.0001
θ	1	.4376	3.6255	.0677
γ	1	1.8074	11.6189	.0021
β	1	.0164	.0608	.8072
α	1	.0164	.1180	.7339

Degrees of freedom for error = 27

types of measurement problems for Equivalence and Nonequivalence II relations or between the measurement problems with distinguishably different units and the conservation problems for any of the three

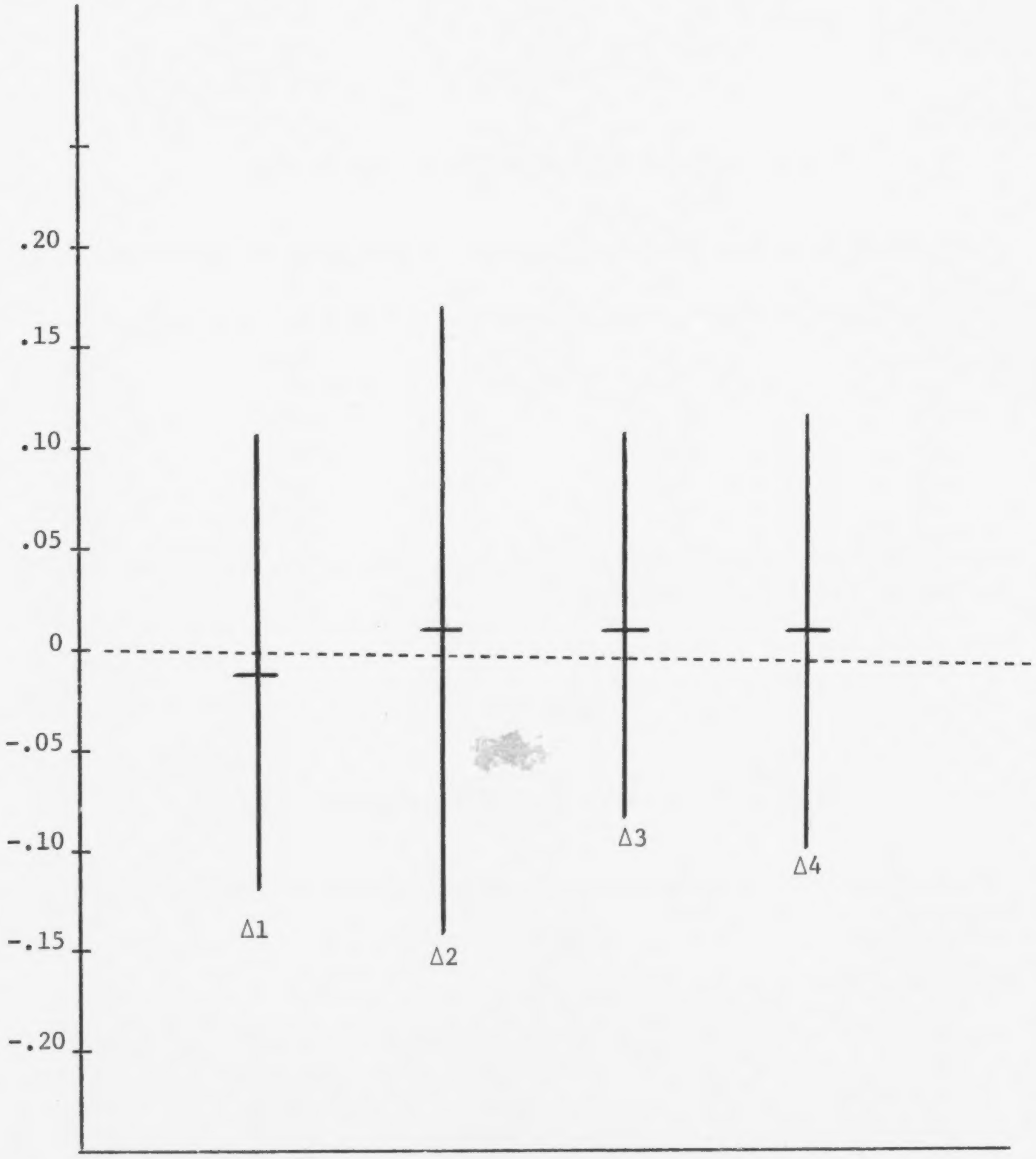


Fig. 7. Confidence Intervals for Deltas in Study IIA

relations. Confidence intervals for the parameters are plotted in Figure 8.

The analysis summarized in Tables 20 and 21 indicates that there

Table 20

MANOVA--Combined Analysis of Sex, Grade, Order of Items, and Interactions for Parameters of the Model for Study IIA

Source	df	MS	F	$p<$
Multivariate	55,110.0488		1.4481	.0511
μ	11	.3689	3.6539	.0030
Θ	11	.1018	.8437	.6009
γ	11	.0914	.5877	.8220
β	11	.4223	1.5655	.1611
α	11	.1321	.9508	.5105

Degrees of freedom for error = 27

Table 21

MANOVA--Combined Analysis of Sex, Grade, Order of Items, and Interactions for Deltas in Study IIA

Source	df	MS	F	$p<$
Multivariate	48,94.4892		1.0477	.4157
$\Delta 1$	12	.2933	1.7597	.1085
$\Delta 2$	12	.2393	.6570	.7754
$\Delta 3$	12	.0448	.2793	.9881
$\Delta 4$	12	.3924	1.9863	.0678

Degrees of freedom for error = 27

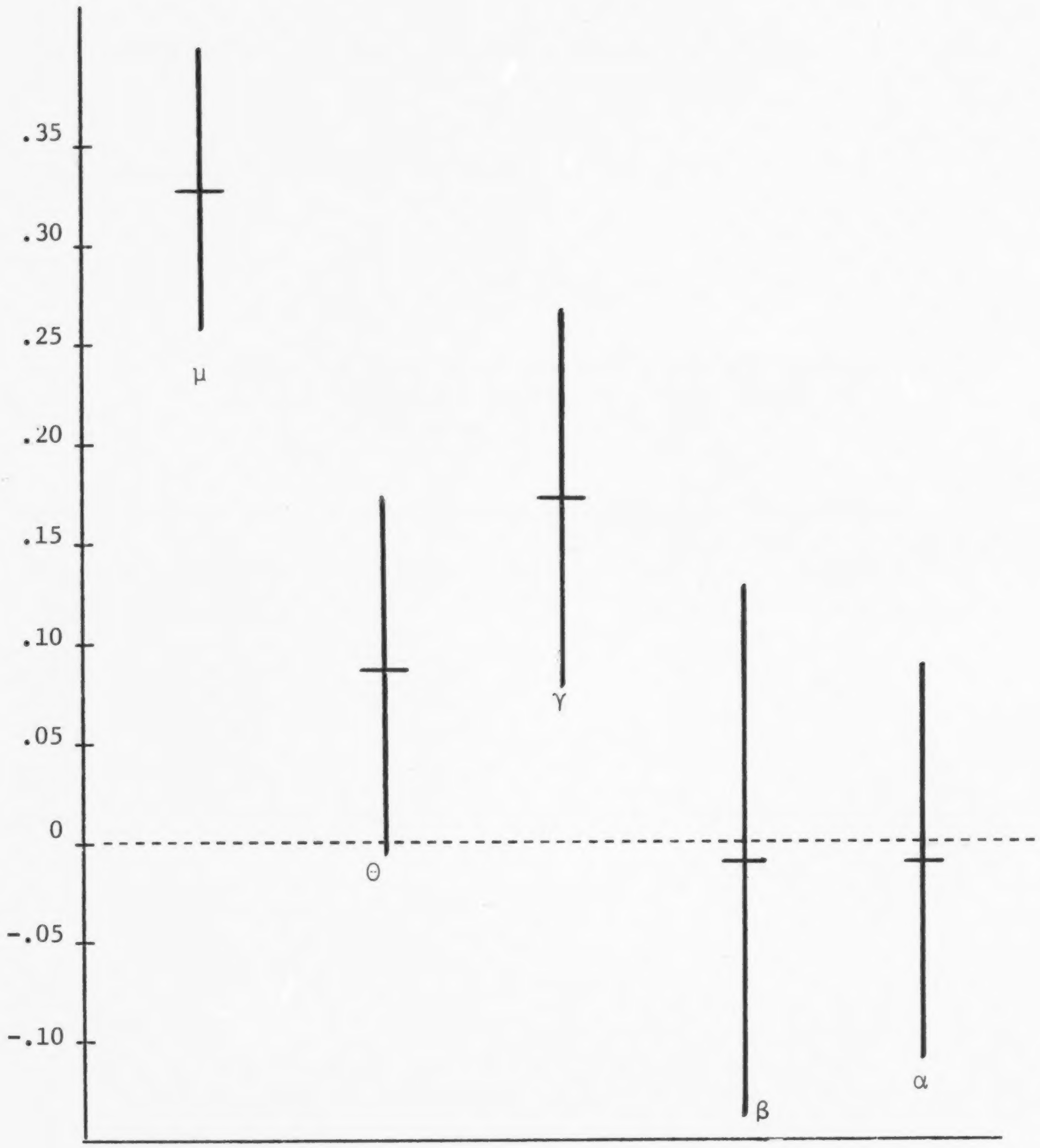


Fig. 8. Confidence Intervals for Parameters in Study IIA

are significant differences for μ due either to grade level, sex, order of the items or some interaction between these variables. There are no significant differences due to these variables for any of the parameters that provide contrasts between problems. Further analysis (Table 22) indicates that the differences for μ are due to grade level.

Table 22

MANOVA--Grade for Parameters of the Model for Study IIA

Source	df	MS	F	p<
Multivariate	5,23		4.9790	.0032
μ	1	2.2930	22.7121	.0001
Θ	1	.0007	.0055	.9414
γ	1	.1058	.6800	.4169
β	1	.0861	.3193	.5768
α	1	.0045	.0322	.8590

Degrees of freedom for error = 27

Study IIB

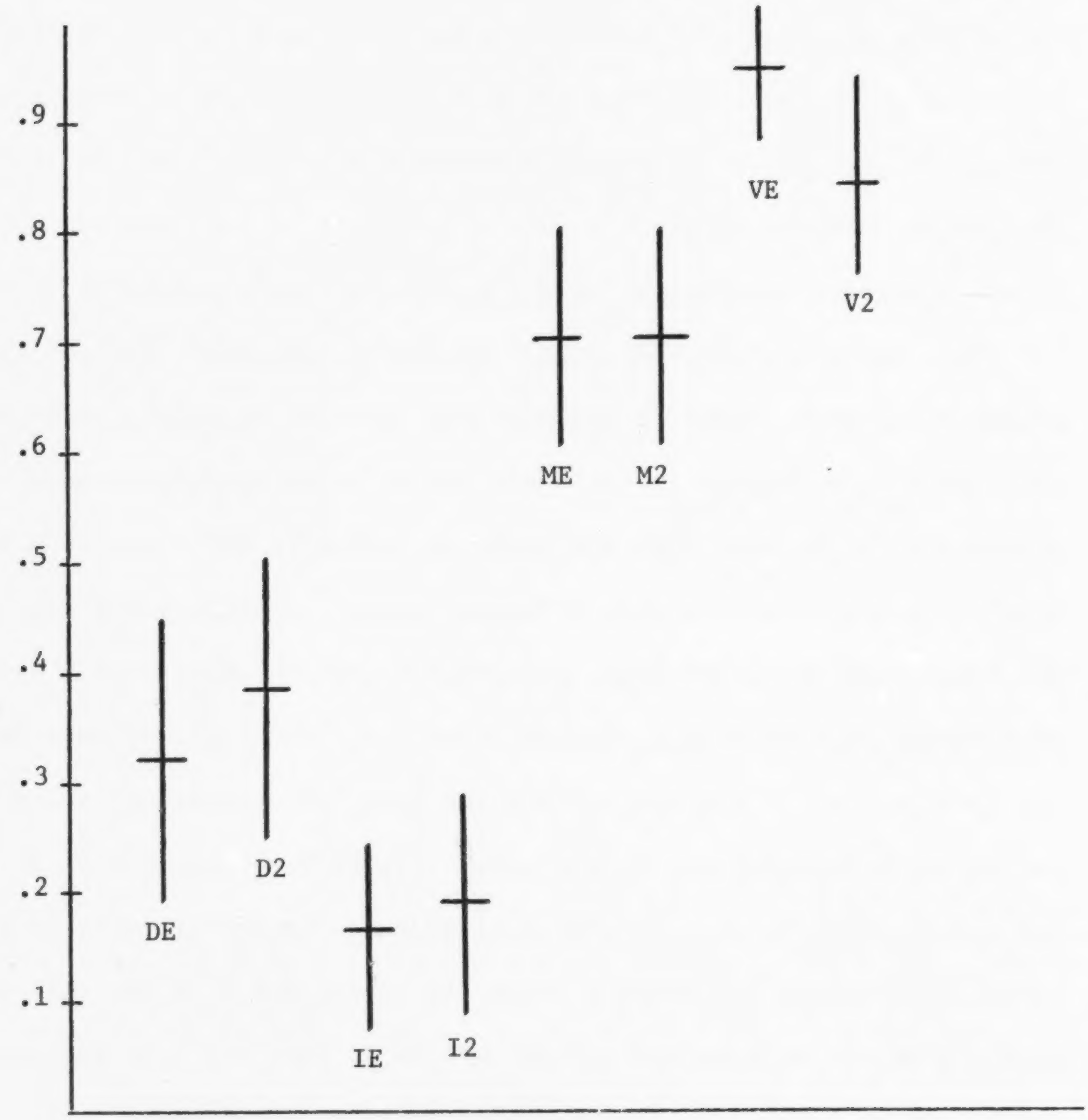
The results of individual items in Study IIB, the reasons given for responses, and the types of errors are summarized in Table 23; and the means for individual items surrounded by 95% confidence intervals have been plotted in Figure 9.

Table 23

Number of Subjects in the Major Response Categories in Study IIB

Item	DE	D2	IE	I2	ME	M2	VE	V2
Total correct	22	26	11	13	47	48	64	58
Reason for correct response								
Reversibility	0	0	0	0	0	0	0	0
Statement of operation performed	0	0	0	0	0	0	0	0
Addition-subtraction	0	0	0	0	0	0	0	0
Compensation, proportionality	5	10	0	0	0	0	0	0
Sameness of quantity	0	0	1	0	0	0	0	0
Reference to previous state	14	11	6	8	45	46	59	55
No reason given or unclassifiable reason given	3	5	4	5	2	2	5	3
Total incorrect	46	42	57	55	21	20	4	10
Type of error								
Taller container or greater number of units	46	40	57	53	21	17	4	7
Wider container or larger unit	0	2	0	2	0	3	0	3

D = Measurement with visibly different units
I = Measurement with indistinguishably different units
M = Measurement with the same unit into apparent inequality
V = Measurement of unequal appearing quantities with the same unit
E = Equivalence
2 = Nonequivalence II



D = Measurement with visibly different units
I = Measurement with indistinguishably different units
M = Measurement with the same unit into apparent inequality
V = Measurement of unequal appearing quantities with the same unit
E = Equivalence
2 = Nonequivalence II

Fig. 9. Confidence Intervals for Items in Study IIB

As in Study IIA virtually all of the reasons for correct responses fall into two categories; and except for the problems in which quantities were measured with distinguishably different units, virtually all the correct responses were based on reference to the previous state. Two of the five Ss who correctly answered both of the problems in which the larger unit was distinguishable but neither of the problems in which it was not gave compensation as the reason for their correct responses.

Only one S missed every item. Another S completely ignored the number cues, even though he successfully counted the number of units; consequently, he missed all the problems in which quantities were measured with the same unit but answered correctly the items in which quantities were measured with different units. A third S who was in the "more-same" protocol group responded "same" to every item. On the measurement problem with indistinguishably different units, only two of the Ss were able to use the information from the measurement operation to correctly identify the larger unit. The rest were unable to apply the inverse relationship between unit size and number of units to this problem and simply responded incorrectly on the basis of the unit that looked larger. Between 85% and 89% of the Ss gave the same response to corresponding Equivalence and Nonequivalence II problems.

The test of the proposed model (See Table 7) is summarized in Tables 24 and 25. The multivariate analysis indicates that the model is appropriate; however, the univariate analysis indicates that there is a significant difference between Equivalence and Nonequivalence II for the

Table 24

ANOVA--Lack of Fit of Model for Study IIB

Source	df	MS	F	p<
Between	1	.0000	.0000	1.0000
Within Cells	40	.0742		

Table 25

MANOVA--Lack of Fit of Model for Study IIB

Source	df	MS	F	p<
Multivariate	3,38		1.6369	.1970
$\Delta 2$	1	.2353	1.9335	.1721
$\Delta 3$	1	.0000	.0000	1.0000
$\Delta 4$	1	.5294	4.6109	.0379

Degrees of freedom for error = 40

problems in which unequal appearing quantities are measured with the same unit of measure. Ninety-five per cent confidence intervals for the Δs have been plotted in Figure 10.

Analysis for the parameters of the model is summarized in Table 26,

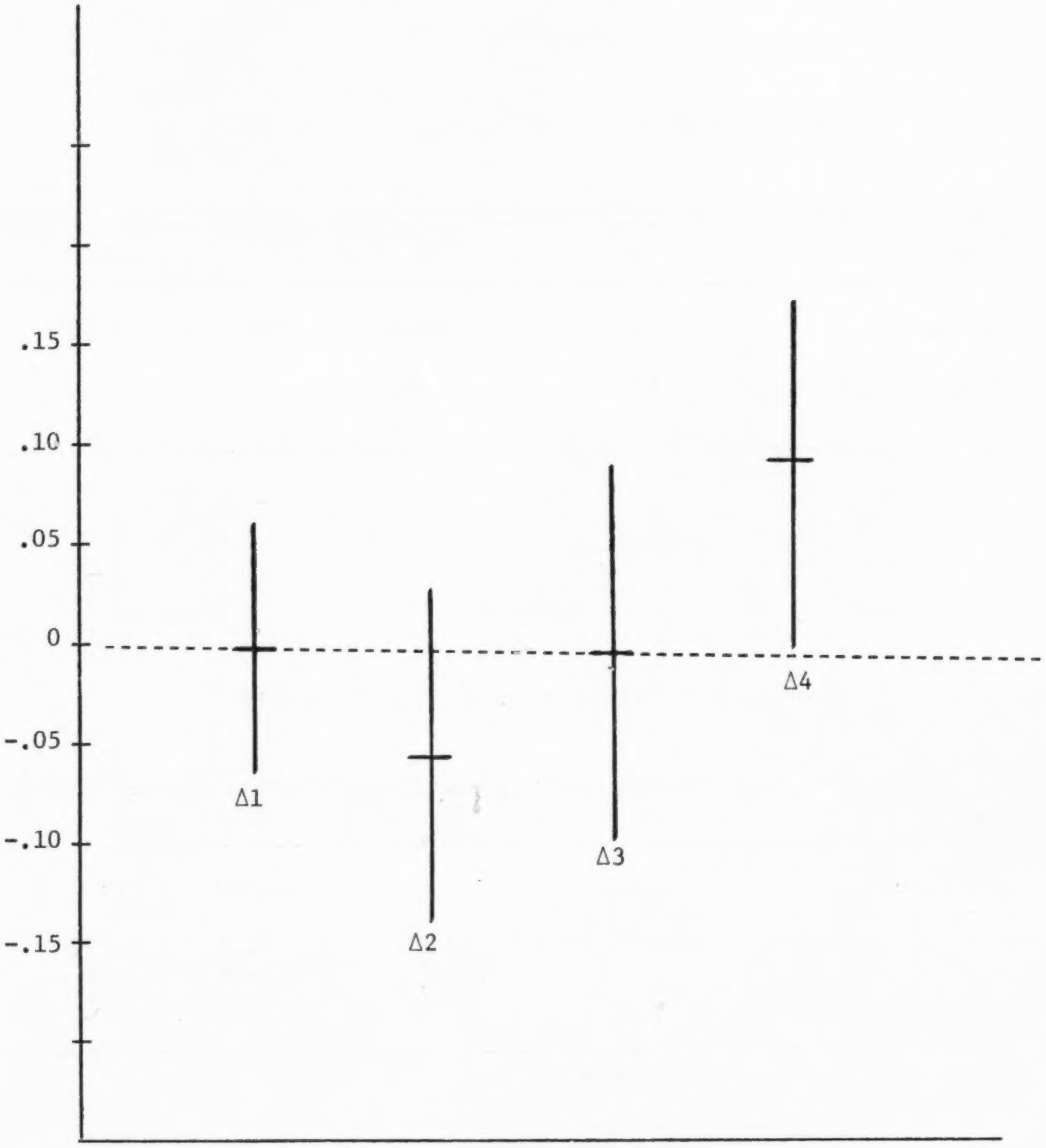


Fig. 10. Confidence Intervals for Deltas in Study IIB

Table 26

MANOVA--Parameters of Model for Study IIB

Source	df	MS	F	p<
Multivariate	3,38		70.8403	.0001
θ	1	2.1176	20.3702	.0001
γ	1	8.4706	33.6859	.0001
α	1	2.4853	19.5404	.0001

Degrees of freedom for error = 40

and confidence intervals for the parameters are plotted in Figure 11. These results indicate that there are significant differences between each of the four types of measurement problems in Study IIB.

The analysis summarized in Tables 27, 28, 29, 30, 31, and 32 indicates that there are significant differences for γ due to sex and order of the items. Reanalysis of the parameters with sex and order effects removed (Table 33) indicates that γ is significantly different from zero irrespective of sex and order effects.

Study II: A and B Comparisons

Four items were given in both parts of Study II. The 95% confidence intervals for corresponding items do intersect (Figure 12); however, comparison of corresponding item means indicates that the fact that differences between measurement problems using visibly different units and

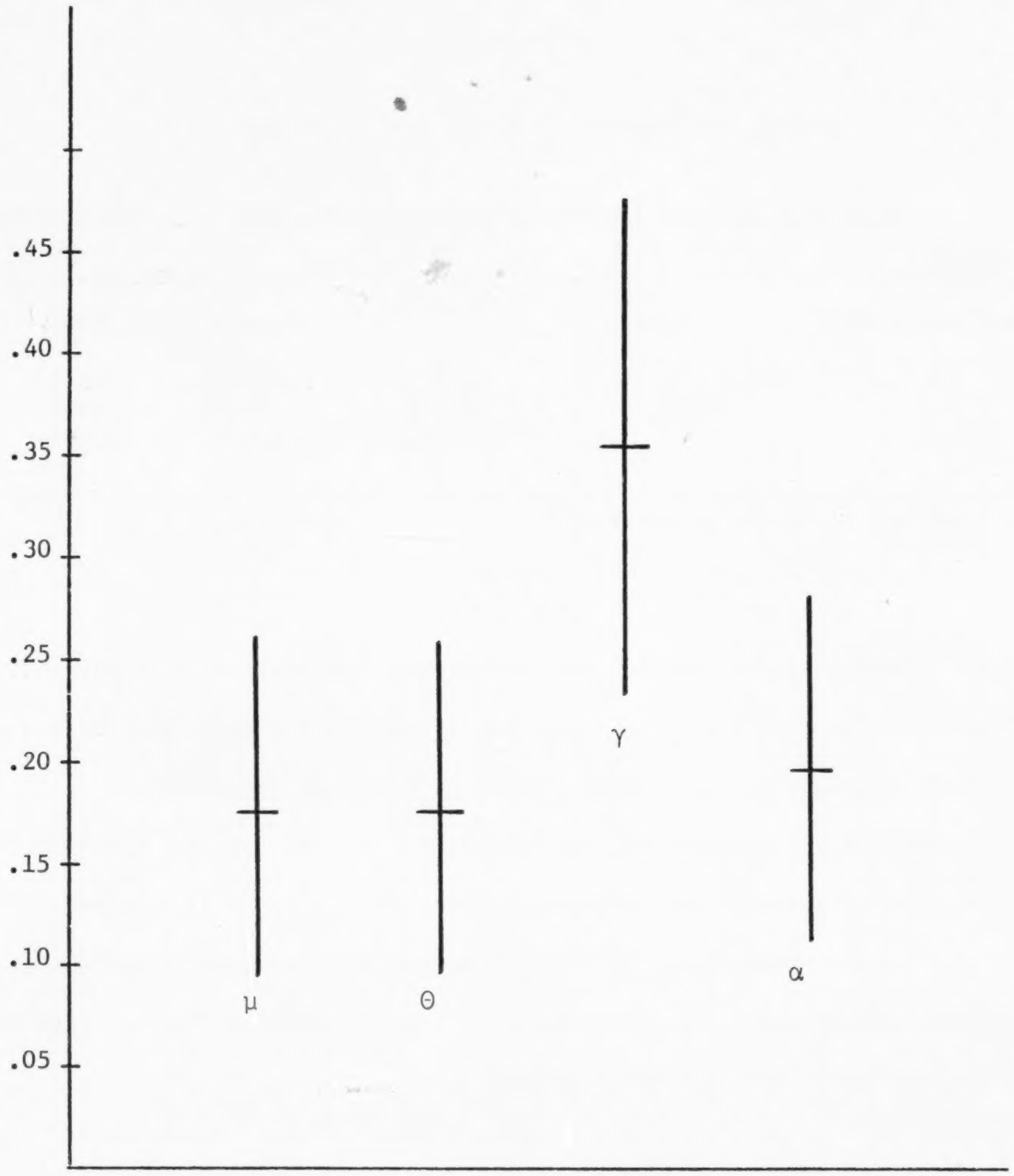


Fig. 11. Confidence Intervals for Parameters in Study IIB

Table 27

ANOVA--Combined Analysis of Grade, Sex, and Order of Items for Delta 1 in Study IIB

Source	df	MS	F	p<
Between	11	.0429	.5783	.8348
Within Cell	40	.0742		

Table 28

MANOVA--Combined Analysis of Grade, Sex, and Order of Items for Deltas in Study IIB

Source	df	MS	F	p<
Multivariate	33,112.6590		.8303	.7255
$\Delta 2$	11	.1749	1.4372	.1946
$\Delta 3$	11	.0778	.5921	.8238
$\Delta 4$	11	.1231	1.0723	.0467

Degrees of freedom for error = 40

Table 29

ANOVA--Combined Analysis of Grade, Sex, and Order of Items for Mu in Study IIB

Source	df	MS	F	p<
Between	10	.1491	1.3696	.2293
Within Cells	40	.1089		

Table 30

MANOVA--Combined Analysis of Grade, Sex, and Order of Items for Parameters in Study IIB

Source	df	MS	F	p<
Multivariate	30,112.2153		1.5930	.0427
θ	10	.1348	1.2963	.2655
γ	10	.7260	2.8872	.0083
α	10	.2093	1.6440	.1293

Degrees of freedom for error = 40

Table 31

MANOVA--Sex for Parameters in Study IIB

Source	df	MS	F	p<
Multivariate	3,38		2.2291	.1006
Θ	1	.2001	1.9340	.1721
γ	1	1.4830	5.8976	.0198
α	1	.0001	.0004	.9841

Degrees of freedom for error = 40

Table 32

MANOVA--Order of Items for Parameters in Study IIB

Source	df	MS	F	p<
Multivariate	21,109.6656		1.5664	.0710
Θ	7	.0958	.9215	.5004
γ	7	.7302	2.9039	.0150
α	7	.2239	1.7608	.1226

Degrees of freedom for error = 40

Table 33

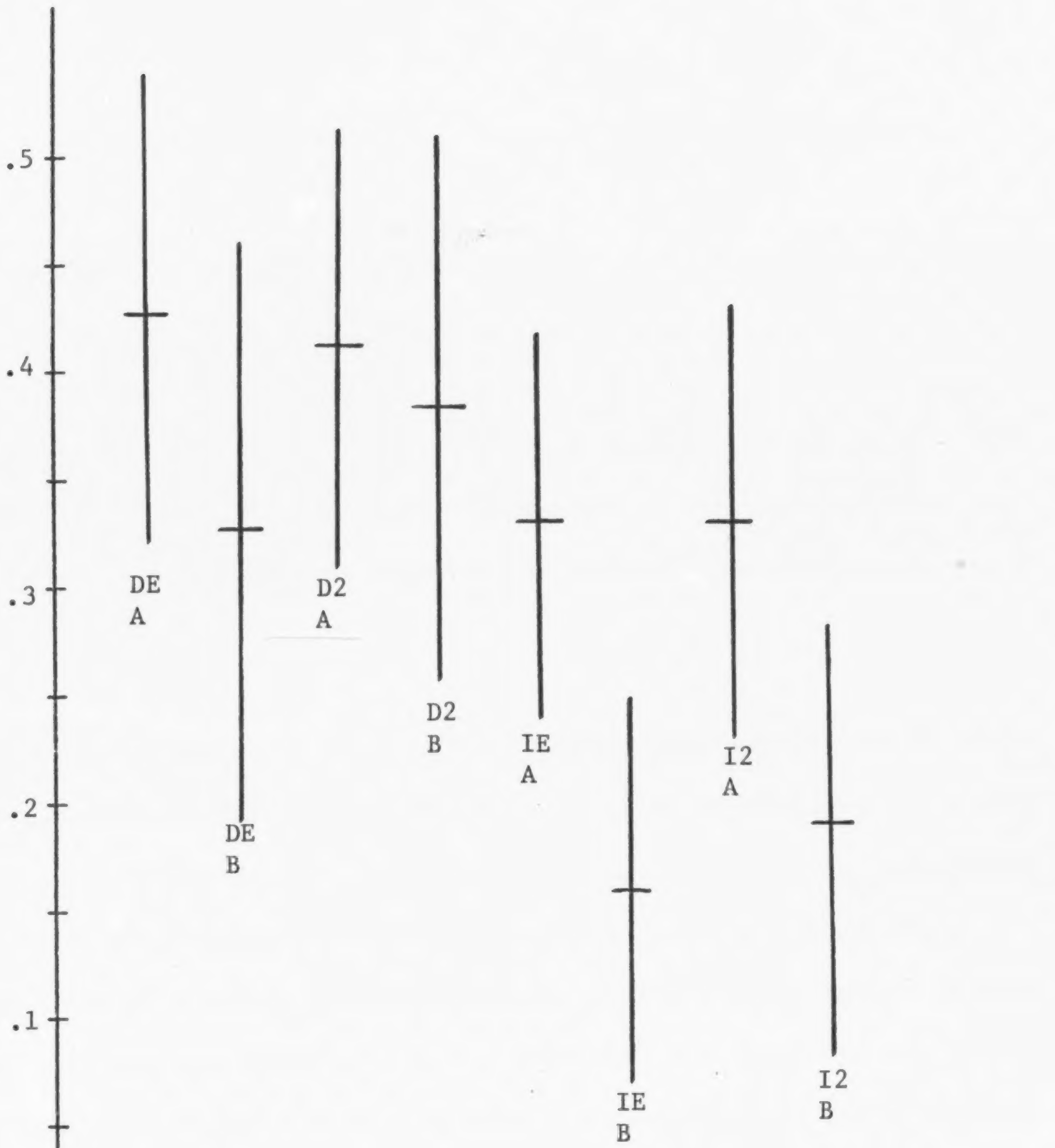
MANOVA--Parameters of Model for Study IIB with Sex and Order Effects Removed

Source	df	MS	F	p<
Multivariate	3,38		45.0230	.0001
Θ	1	1.1946	11.4910	.0016
γ	1	5.6515	22.4748	.0001
α	1	1.6393	12.8892	.0009

Degrees of freedom for error = 40

those using indistinguishably different units are significant in Part B but fail to reach significance in Part A can be attributed entirely to between study differences in performance on the problems employing indistinguishably different units.

Analysis of variance for contrasts between conservation of continuous quantity and measurement problems in which quantities were measured with the same unit into apparent inequality (i.e. the final state of the conservation problems) is summarized in Table 34 indicating significant differences favoring the measurement problems. These results should be interpreted somewhat cautiously in that the two types of problems were administered in different sets of problems in the series. The results in Figure 12, however, indicate that for the four problems that were administered in both studies, performance was generally higher in the



D = Measurement with visibly different units
I = Measurement with indistinguishably different units
E = Equivalence
2 = Nonequivalence II

Fig. 12. Confidence Intervals for Items Appearing in Both Study IIA and IIB

Table 34

ANOVA--Conservation of Continuous Quantity-Measurement into Apparent Inequality Contrast

Source	df	MS	F
Between	1	11	14.6**
Within Cells	127	.76	

**p < .01

study containing the conservation problems; so the danger of interaction with other problems favoring the measurement problems is probably not too great.

Furthermore, in Study IIA no significant difference was found between conservation problems and corresponding problems in which quantities are measured with different units. In Study IIB problems in which quantities are measured with the same unit were found to be significantly easier than corresponding problems in which two units are employed. The combination of these results confirms that problems in which quantities are measured with the same unit are easier than corresponding conservation problems.

Chapter V

DISCUSSION

Summary and Conclusions

This study was designed to investigate the development of certain measurement concepts, to relate this development to the development of conservation, and to determine the role of equivalence and nonequivalence relations in conservation and measurement problems.

Two hundred eighteen Ss in grades K--2 were individually tested in two studies each of which was divided into two parts. A total of eighteen conservation and measurement items were administered in the complete investigation. The conservation problems were the classical continuous quantity and discrete object problems. In one set of measurement problems, quantities were measured with different size units of measure. Centering on either unit size or the number of units alone lead to errors similar to those found in conservation problems. In a second set of measurement problems, quantities in different shaped containers were measured with the same unit. Each type of problem was administered in three situations employing different combinations of equivalence and nonequivalence relations.

The results of the complete investigation are summarized and interpreted below. The symbols in parentheses refer to the specific study in which the results appeared.

Equivalence and nonequivalence relations in conservation and measurement problems. 1) In general there does not appear to be any significant difference between conservation and measurement problems employing Equivalence relations and corresponding problems employing Nonequivalence II relations (IA, IB, IIA, IIB).

Significant differences favoring Equivalence were found, however, in the problems in which unequal appearing quantities were measured with the same unit of measure (IIB). Since the correct cues appeared last, the correct answer was the most natural; and it is curious that there was even a significant number of incorrect responses much less a significant difference between relations. These results should be regarded with some caution since Equivalence-Nonequivalence II contrasts did not even approach significance for any of the other types of problems. The probability of a Type I error on one of the nine Equivalence-Nonequivalence II contrasts may be as high as .45.

2) Nonequivalence I problems are significantly easier than corresponding problems employing Equivalence or Nonequivalence II relations except in problems in which it is not possible to identify the larger unit (IIB). These results imply that the relation between quantities being compared does not affect performance, and the Nonequivalence I are easier simply because they do not require genuine conservation since accurate comparisons can be made from the final states of the quantities.

Development of measurement concepts. 1) Measurement operations are meaningful for the vast majority of students in the first and second grades. By the end of the first grade, virtually all students realize that the quantity that measures the most units must be greatest. Only 3 of the 129 Ss tested did not respond to any questions on the basis of measurement cues; and only 2 of the 3 definitely ignored the measurement cues. The other S simply responded "same" to all problems (IIA, IIB).

This does not mean, however, that first and second grade students have accurate measurement concepts or are able to accurately apply measurement processes. Only 70% of the Ss tested were able to use measurement results if they were followed by conflicting visual cues (IIB). Only 59% of the Ss tested demonstrated any knowledge that variations in unit size affected measurement results (IIA), and as few as 40% of the Ss were able to apply this knowledge to problems in which quantities were measured with different units (IIA, IIB). This figure dropped to 25% of the Ss when the larger unit was not visibly distinguishable, and only 6% of the Ss were able to use results of measurement operations to determine the larger unit when it was not visually apparent.

The conclusion that by the end of first grade virtually all children, even those in stages I and IIA, have some concept of measurement appears to contradict Piaget's (1960) conclusion that measurement concepts do not begin to appear until stage IIB. This apparent conflict is due to the fact that Piaget employed less structured measurement tasks. In order to have any measurement cues to respond to, Ss had to measure themselves. In the current investigation the measurement cues were forced upon the Ss; therefore, even Ss in the earliest stages had number cues to guide or distract their response.

2) There is no significant difference between conservation problems and corresponding measurement problems in which the distracting cues are numerical (IIA). Piaget (1952, 1960) as well as most of those replicating and extending his studies have investigated conservation using tasks that involved visual distortion and have described the earliest stages in the development of conservation and measurement as being dominated by perceptual judgements. Bruner has taken the most extreme position in this regard. He asserts that young children are highly dependent on perceptual properties of events. Conservation errors occur because the immediate perceptual properties of the conservation problems override the logical properties that imply conservation. Thus, for Bruner conservation failures occur because of the "perceptual seduction" (Bruner, Olver, & Greenfield, 1966, p. 192) inherent in the conservation problems. The results of the current investigation, however, demonstrate that misleading numerical cues produce the same errors as misleading visual cues. Thus, it appears that it is not simply the perceptual properties of the stimuli that produce errors in conservation problems.

3) The failure of young children to respond primarily on the basis of visual cues is even more striking in the contrast between conservation problems and the problems in which quantities are measured into apparent inequality and the contrast between the problems in which quantities are measured with distinguishably different units and the problems in which quantities are measured into apparent inequality. The problems measuring quantities into apparent inequality, in which correct measure-

ment cues are followed by misleading perceptual cues, are significantly easier than either corresponding conservation problems, in which both sets of cues are visual, or corresponding problems in which quantities are measured with different units, where correct visual cues are followed by incorrect numerical cues (IIA, IIB).

These results, which could be interpreted to imply that numerical modes dominate visual modes, should be regarded with some caution. Zimiles (1963) has suggested that conservation failures may result from Ss basing their judgements on the E's manipulations of the quantities being compared. For example, if two rows of blocks which the S has judged equivalent when they are arranged in one-to-one correspondence are spread out, the S says that the longer row has more because the act of spreading the blocks out implies to him that the length of the rows is the dimension he is being asked to compare.

In the current investigation, the experimental procedures emphasize the measurement cues, which means that the correct choice is emphasized in the problems employing a single unit of measure but the incorrect choice is emphasized in the problems employing different units of measure and the conservation problems.

4) Problems in which correct cues appear last are significantly easier than corresponding problems in which correct cues are followed by misleading cues (IIB). As noted in (3) above, however, the order of the cues was not the only factor that was found to affect responses.

5) The contrasts between measurement problems in which it is possible to distinguish the large unit and those in which it is not

are ambiguous. Significant differences between the two problems were found in Study IIB but not in Study IIA. In Study IIA, however, about 7% of the Ss tested did find the problems in which the larger unit was distinguishable easier than the problems in which it was not. Consideration of this fact and examination of the confidence intervals for the parameter Θ indicates that probably at least 10% of the population sampled require that the distracting cues contain compensating relations in order to conserve.

6) Although comparisons between different investigations must be regarded with caution due to different populations, different experimental procedures, etc., several comparisons between the Carpenter (in press) study and the current investigation are worth mentioning. In the earlier study 7 of the 16 Ss (44%) who missed Item E, in which quantities were measured with different spoons, chose the quantity measured by the larger spoon as having more rice whereas in the current investigation only 2 of the 71 incorrect responses to the corresponding item were due to Ss' choosing the larger unit.

Second, the magnitude of the difference between Equivalence and Nonequivalence I problems is much greater in the earlier study, in which 65% of the Ss correctly answered the Nonequivalence I problem but missed the Equivalence problem as opposed to a difference of only 16% in the current investigation. These differences seem to imply that variations in units are more apparent to young children for spoons measuring rice than for glasses measuring water.

Contrasts between different conservation situations. 1) Significant differences exist between conservation of continuous quantity problems and conservation problems with discrete objects (IB).

2) Conservation problems in which discrete objects were counted were significantly easier than corresponding problems in which the objects were not counted; however, the magnitude of the difference was surprisingly small, with less than 5% of the population performing better on the problems in which the objects were counted (IA). These results confirm the findings of Wohlwill and Lowe (1962), who found that only 4 of 23 nonconservers changed to conservation responses when asked to count the arrays of objects being compared after making nonconservation responses, but at best provide weak support for Zimiles' (1963) hypothesis that inducing a numerical set would significantly improve conservation performance.

Experimental and population variables. 1) By the end of kindergarten and the first grade few children still respond to conservation problems on the basis of the last alternative offered to them. Only one S in the entire investigation consistently responded either "more" or "same" to all problems (IA, IB, IIA, IIB).

2) Significant differences due to sex and order of items were found for the parameter contrasting the problems in which quantities are measured with distinguishably different units and the problems in which quantities are measured with the same unit into apparent nonequivalence (IIB). It is difficult to explain why these variables are significant for this parameter but not for any of the others in the

investigation.

Implications for Instruction

This study has been conducted in connection with the mathematics development project Developing Mathematical Processes (DMP). A professed goal of this project is "to explain fundamental learning processes associated with [the topics in the mathematics program]." (Harvey, Romberg, & Fletcher, 1969) This study provides a detailed description of young children's understanding of certain concepts basic to the measurement process. If the curriculum is to be geared to the abilities of the learner and build on the knowledge that he already possesses, this study should provide valuable guidelines for teachers and curriculum developers who, like the DMP mathematics program, are attempting to introduce measurement in the early grades.

Caution must be exercised in applying the results of this study or any other status study to the curriculum. Status studies investigate what children have learned in a given environment. They do not demonstrate what children are capable of learning if a different set of experiences are provided. On the other hand performance on Piagetan type tasks has not proved to be susceptible to change by variations in instruction.

Second, the current investigation and related studies do not dictate specific programs of instruction. They simply provide guidelines within which instruction should fall. Instruction should agree with the results of these studies but there is not necessarily just one kind of instruction that would so agree.

For example, the fact that no differences were found between the way students respond to equivalence and nonequivalence relations supports the procedure employed by DMP of integrating their instruction; however, these results do not dictate this as the only logical sequence. On the other hand this knowledge that the relations develop concurrently and that one does not depend on the other should prove valuable in structuring activities and also should allow for more flexibility in assessment.

Measurement processes are meaningful for the majority of students in the first and second grades. By the end of the first grade, virtually all students realize that numbers can be assigned to quantities and can be used to compare the quantities with the greater quantity measuring the greater number of units. Certainly a number of the most elementary measurement concepts can be taught meaningfully to young children; however, several basic measurement misconceptions have been identified that appear to be linked to basic logical structures of the child.

It appears that in order to overcome some of the measurement misconceptions that have been identified it is not sufficient to simply provide more practice with measurement processes. Many measurement experiences, especially those dealing with different units of measure, do not have the same meaning for young children that they have for adults. Many young children do not recognize that a conflict exists when quantities that were equal measure different numbers of units. Unless this tendency to focus on immediate dominant dimensions is overcome, it would appear to be very difficult to convince a child of the need for a constant unit of measure.

These results do not imply that experiences with different units of measure should not be included in measurement topics. Such experiences may help young children to expand their logical framework. They do imply, however, that many young children will not master all the implications of different units by concentrating on measurement processes. If one is really concerned with mastery of measurement concepts with different units of measure, it would seem necessary to provide a wide range of experiences that help a child to focus on more than one immediate dominant dimension. It is important for teachers and curriculum developers to know when they are providing experiences that can be mastered, when they are providing experiences that may be learned superficially, and when they are providing experiences that may be beyond the capabilities of many of the children.

It is clear from the results of this study that a wide range of tasks are necessary to adequately assess the development of measurement concepts. The tasks in this study provide models for types of tasks that assess some of the misconceptions identified in this study.

Implications for Future Research

1) This study seems to have adequately answered in the negative whether differences in relations involved in conservation and measurement problems affect performance. A wide range of tasks were given to a reasonably large sample of Ss, and in general no significant differ-

ences were found that could be attributed to the relations. It might be worthwhile, however, to replicate the contrast between relations in problems in which apparently unequal quantities are measured with the same unit in order to determine whether the significant differences found for these problems really do represent spurious significance.

2) The problems from this study should be administered to different samples from different populations in order to test the generalizability of the results and the extent to which they are dependent on instruction. It would be informative to determine whether students in the DMP program, who have studied an integrated approach to number and measurement concepts, would make the same errors as the students tested in this investigation, who studied a more traditional mathematics program. Does the apparent predisposition to favor numerical cues result from the emphasis on number in the traditional curriculum?

3) The generalizability of the results to length and area concepts might also be tested. The comparison of the results of the current investigation with the earlier Carpenter (in press) study indicates that interesting differences may even exist within volume comparisons in which different materials and measuring instruments are used.

4) A longitudinal study with repeated measures on the same _S_s should also be conducted in order to determine whether the differences found represent developmental stages or are simply caused by differences in difficulty of the problems.

5) An extension of the current investigation should be conducted in which Ss are asked to predict the number of units the second quantity measured will measure. In the Equivalence problem a number of Ss who subsequently missed the problem predicted that the quantity measured with the larger unit would measure fewer units. Items in which Ss are asked to predict may demonstrate that a greater number of young children actually understand the inverse relationship between unit size and number of units than would be predicted from the results of this study.

Appendix I

SUBJECT DATA FOR STUDY I

Subject Data for Study I

Codes: Subj. char.--PQR: P = Teacher: 1 or 2
Q = Morning or Afternoon
R = Male or Female

Expt. var.--STU: S = Order group (Tables 2 and 3)
T = Protocol order:"More-same" or "Same-more"
U = Array Toward S or Away from S is moved

Subj.	Subj. char.	Expt. var.	Age in mos.	Subj.	Subj. char.	Expt. var.	Age in mos.
Part A							
63	2MM	2MA	68	32	1AF	1ST	71
36	1AM	2MA	68	18	1MM	1ST	77
59	2MM	2MA	74	76	2AF	1ST	71
13	1MM	2MA	70	88	2AF	1ST	77
87	2MF	2MA	74	93	2AM	1ST	70
51	2MF	2ST	74	48	1AF	1SA	67
69	2MM	2ST	77	28	1AM	1SA	73
29	1AM	2ST	68	49	1AM	1SA	69
2	1MM	2ST	69	81	2AM	1SA	75
73	2MM	2SA	72	85	2AF	1SA	70
12	1MF	2SA	70	38	1AF	2MT	66
61	2MM	2SA	73	35	1AM	2MT	70
55	2MM	2SA	67	5	1MM	2MT	77
94	2AF	2SA	74	82	2AM	2MT	78
84	2AM	3MT	71	37	1AM	2MT	77
79	2AM	3MT	74	62	2MF	3SA	76
40	1AF	3MT	74	6	1MF	3SA	74
60	2MF	3MT	70	68	2MF	4MT	67
70	2MF	3MA	69	47	1AM	4MT	73
20	1MM	3MA	71	17	1MM	4MT	71
23	1MM	3ST	73	53	2MM	4MT	75
95	2AM	3ST	72	64	2MF	4MA	72
58	2MM	3ST	69	77	2AM	4MA	76
75	2MM	3ST	73	31	1AF	4MA	77
90	2MA	1MT	71	78	2AM	4MA	77
4	1MF	1MT	72	56	2MF	4ST	71
86	2AF	1MT	74	1	1MF	4ST	74
43	1AM	1MT	74	26	1AF	4ST	71
9	1MF	1MT	74	25	1MM	4SA	68
54	2MF	1MA	78	27	1AM	4SA	68
15	1MM	1MA	74	14	1MF	3SA	75
33	1AF	1MA	70	3	1MM	1MA	72
11	1MM	1MA	70				

Subject Data for Study I (continued)

Subj.	Subj. char.	Expt. var.	Age in mos.	Subj.	Subj. char.	Expt. var.	Age in mos.
Part B							
71	2MM	3MT	74	41	1AM	4MT	68
57	2MM	1MT	74	19	1MF	2MA	76
65	2MF	1MA	76	34	1AF	4MA	69
52	2MM	3MA	75	21	1MM	2MA	71
39	1AF	3MA	75	83	2AM	2MA	77
46	1AF	3MA	71	16	1MM	2ST	78
96	2AF	3ST	73	10	1MM	4ST	78
80	2AF	1ST	74	30	1AM	2ST	77
74	2MF	1SA	71	22	1MF	4ST	77
92	2AM	2MT	75	45	1AM	4SA	68
89	2AF	2MT	76	91	2AF	2SA	76
8	1MM	4MT	78	44	1AF	4SA	83

Appendix II

CLASSIFICATION OF RESPONSES FOR STUDY IA

Table II-1 Subject Performance on Items in Study IA

Table II-2 Subjects' Reasons for Correct Responses and Types of Errors in Study IA

Code:

S = Conservation of discontinuous quantity
N = Conservation of discontinuous quantity with counting
E = Equivalence
2 = Nonequivalence II

Table II-1

Subject Performance on Items in Study IA

Code: + Correct
0 Incorrect

Subj.	SE	S2	NE	N2	SER	S2R	NER	N2R	Total correct
63	0	0	+	+		0			2
36	0	0	0	0		0			0
59	0	0	+	+		0			2
13	0	0	0	0		0			0
87	0	0	0	0		0			0
51	0	+	0	0		0			1
69	+	+	+	+		+			5
29	0	0	0	0		0			0
2	0	0	0	+		0			1
73	+	+	+	+		+			5
12	0	0	0	0		0			0
61	+	+	+	+		+			5
55	+	+	+	+		+			5
94	0	0	0	0		0			0
84	+	+	+	+			+		5
79	+	+	+	+			+		5
40	+	+	+	+			+		5
60	+	+	0	+			+		4
70	0	0	0	0			0		0
20	+	+	+	+			+		5
23	+	+	+	+			+		5
95	0	0	0	0			0		0
58	0	0	0	0			0		0
75	+	+	+	+			+		5
90	+	+	+	+	+				5
4	+	+	+	+	+				5
86	+	+	+	+	+				5
43	+	+	+	+	+				5
9	+	+	+	+	+				5
54	+	+	+	+	+				5
15	0	0	0	0	0				0
33	0	0	0	0	0				0
11	+	+	+	+	+				5
3	+	+	+	+	+				5
32	0	0	0	0	0				0
18	+	+	+	+	+				5
76	+	+	+	+	+				5

Table II-1 (continued)

Subj.	SE	S2	NE	N2	SER	S2R	NER	N2R	Total correct
88	0	0	0	0	0				0
93	0	0	0	0	0				0
48	0	0	0	0	0				0
28	+	+	+	+	+				5
49	+	+	+	+	+				5
81	0	0	0	0	0				0
85	0	+	+	+	+				4
38	0	+	0	0		0			1
35	0	0	0	0		0			0
5	+	+	+	+		+			5
82	+	+	+	+		+			5
37	+	+	+	+		+			5
62	0	0	0	0			0		0
6	+	+	+	+			+		5
68	0	0	0	0				0	0
47	0	+	0	0				0	1
17	+	+	+	+				+	5
53	0	0	0	0				0	0
64	0	0	0	+				0	1
77	+	+	+	+				+	5
31	0	0	0	0				0	0
78	+	+	+	+				+	5
56	+	+	+	0				+	4
1	0	0	0	0				0	0
26	+	+	+	+				+	5
25	0	0	0	0				0	0
27	0	0	0	0				0	0
14	0	0	0	0			0		0

Table II-2

Subjects' Reasons for Correct Responses and Types of Errors in Study IA

Code:
1 Reversibility
2 Statement of operation performed
3 Addition-subtraction
4 Compensation, proportionality
5 Sameness of quantity
6 Reference to previous state
7 No reason given or unclassifiable reason given
8 Longer row
9 Blocks closer together

Subj.	SE	S2	NE	N2	SER	S2R	NER	N2R
62	8	8	8	8			8	
6	3	3	3	3			3	
68	8	8	8	8				8
47	9	3	8	8				9
17	4	4	2	4				4
53	8	8	8	8				8
64	9	9	9	7				9
77	7	7	7	4				7
31	8	8	8	8				8
78	2	2	2	2				2
56	3	3	6	8				3
1	9	9	9	8				9
26	3	3	3	4				3
25	8	8	8	8				8
27	8	8	8	8				8
63	8	8	4	7		8		
36	8	8	8	8		8		
59	8	8	3	3		8		
13	8	8	8	8		8		
87	8	8	8	8		8		
51	8	6	8	8		8		
69	2	2	2	2		2		
29	8	8	8	8		8		
2	9	8	8	3		8		
73	6	6	6	6		6		
82	2	7	2	2		2		
37	6	6	6	6		6		
12	8	8	8			8		
61	4	4	4	4		4		
55	4	4	4	4		4		
94	8	8	8	8		8		
84	2	2	2	2			2	
79	4	4	3	2			2	

Table II-2 (continued)

Subj.	SE	S2	NE	N2	SER	S2R	NER	N2R
40	4	4	4	4			4	
69	4	4	8	4			4	
70	8	8	8	8			8	
20	2	2	2	2			2	
23	3	3	3	3			3	
95	8	8	8	8			8	
58	8	8	8	8			8	
75	6	6	6	6			6	
90	2	5	5	5	5			
4	7	7	7	7	7			
86	4	4	4	4	4			
43	7	7	7	7	7			
9	7	7	7	7	7			
54	6	7	2	7	7			
15	9	9	9	9	9			
33	9	9	9	9	9			
11	1	3	3	3	6			
3	3	3	3	3	3			
32	8	8	8	8	8			
18	6	2	2	2	2			
76	6	6	6	6	6			
88	8	8	8	8	8			
93	8	8	8	8	8			
48	8	8	8	8	8			
28	4	4	4	4	4			
49	6	6	3	3	6			
81	8	8	8	8	8			
85	8	6	6	6	4			
38	8	7	8	8		8		
35	8	8	8	8		8		
5	2	2	2	2		2		
14	8	8	8	8			8	

Appendix III

CLASSIFICATION OF RESPONSES FOR STUDY IB

Table III-1 Subject Performance on Items in Study IB

Table III-2 Subjects' Reasons for Correct Responses and Types of Errors in Study IB

Code: C = Conservation of continuous quantity
S = Conservation of discontinuous quantity
E = Equivalence
1 = Nonequivalence I
2 = Nonequivalence II

Table III-1

Subject Performance on Items in Study IB

Code: + Correct
0 Incorrect

Subj.	CE	C2	SE	S1	Total correct
71	0	0	0	0	0
57	0	0	0	0	0
65	+	+	+	+	4
52	0	0	0	0	0
39	0	0	0	0	0
46	0	0	0	+	1
96	0	0	0	0	0
80	0	+	+	+	3
74	0	0	0	+	1
92	0	0	0	0	0
89	0	0	0	0	0
8	0	0	+	+	2
41	0	0	0	0	0
19	0	0	0	0	0
34	0	+	0	0	1
21	0	0	0	0	0
83	+	0	+	+	3
16	0	0	+	+	2
10	0	0	0	0	0
30	+	0	+	+	3
22	0	0	+	+	2
45	0	+	0	0	1
91	0	0	+	+	2
44	0	0	0	0	0

Table III-2

Subjects' Reasons for Correct Responses and Types of Errors in Study IB

Code: 1 Reversibility
2 Statement of operation performed
3 Addition-subtraction
4 Compensation, proportionality
5 Sameness of quantity
6 Reference to previous state
7 No reason given or unclassifiable reason given
8 Length of row or height of water
9 Density of row or width of container

Subj.	CE	C2	SE	S1
89	8	8	8	8
8	8	8	2	7
41	8	8	8	8
19	8	8	8	8
34	8	7	8	8
21	8	8	9	8
83	7	8	4	7
16	8	8	2	2
10	8	8	8	8
30	7	8	3	2
22	8	8	3	3
45	8	6	8	8
91	8	8	3	3
44	8	8	9	9
71	7	7	8	8
57	7	7	8	8
65	6	6	6	6
52	8	7	8	8
39	8	8	8	8
46	8	8	8	4
96	8	8	8	8
80	8	7	6	2
74	8	8	9	7
92	7	7	9	8

Appendix IV

SUBJECT DATA FOR STUDY II

Subject Data for Study II

Codes: Subj. char.--PQR: P = Grade
Q = Teacher: 1, 2, 3, 4, or 5
R = Male or Female

Expt. var.--ST S = Protocol Order: "More-same" or "Same-more"
T = Large unit measured first, Small unit measured first

Subj.	Subj. char.	Expt. var.	Age in mos.	Subj.	Subj. char.	Expt. var.	Age in mos.
Part A							
4	11M	ML	93	75	13F	ML	85
5	11F	SL	96	76	13F	ML	107
6	11M	MS	81	77	24M	SS	91
7	11M	SS	84	78	24M	ML	107
14	11M	ML	83	80	24M	ML	107
16	11M	ML	86	82	24F	MS	111
19	11F	MS	85	86	24F	SS	92
21	11F	SS	89	87	24M	SS	100
22	11M	SS	87	88	24M	SL	97
45	11F	ML	79	90	24F	ML	102
46	11F	SS	85	91	24F	ML	102
47	11M	MS	86	92	24M	SL	104
25	12M	ML	81	93	24F	SL	102
27	12F	ML	85	94	24F	SS	105
35	12F	SS	86	98	24F	SS	99
40	12F	MS	92	100	24F	MS	101
38	12M	SL	106	101	24M	ML	95
42	12M	SL	92	107	25M	SS	102
50	13F	SL	85	104	25M	ML	103
52	13M	ML	85	108	25M	MS	92
53	13F	ML	80	110	25F	SL	100
54	13M	SL	89	111	25F	MS	102
55	13F	MS	86	113	25F	MS	103
56	13M	SS	91	116	25F	MS	98
58	13M	SS	91	121	25F	SS	96
61	13F	MS	90	122	25M	SL	104
64	13M	MS	84	126	25M	ML	102
65	13F	ML	86	127	25M	SL	103
71	13F	SS	92	128	25M	SL	97
72	13F	MS	89	130	25M	SL	104
74	13M	SS	82				

Subject Data for Study II (continued)

Subj.	Subj. char.	Expt. var.	Age in mos.	Subj.	Subj. char.	Expt. var.	Age in mos.
Part B							
1	11M	ML	94	62	13M	SL	92
2	11M	SS	82	63	13M	SL	94
3	11M	ML	91	66	13M	SS	90
8	11M	SS	81	67	13M	SS	90
9	11M	SS	86	68	13M	ML	93
10	11M	SS	79	69	13F	MS	81
11	11M	SL	91	70	13F	SL	82
12	11F	SL	87	73	13F	SS	83
13	11F	SL	80	79	24F	SS	98
15	11F	SS	91	81	24F	MS	104
17	11F	SS	85	84	24M	SS	105
18	11F	SS	92	85	24M	MS	104
20	11F	ML	83	89	24F	ML	101
23	11M	MS	79	95	24M	SS	96
24	11M	ML	82	96	24F	SS	94
48	11M	MS	88	97	24M	SL	102
26	12M	SL	107	99	24M	SS	93
28	12M	SL	93	102	24M	ML	105
29	12M	MS	116	103	24F	SL	100
30	12M	ML	115	106	25M	SL	102
31	12M	ML	89	105	25F	SS	105
32	12F	MS	94	112	25F	SS	95
33	12M	MS	89	114	25M	SL	104
34	12M	SS	105	115	25M	MS	101
37	12F	SL	97	117	25M	ML	101
39	12M	SL	83	118	25F	MS	98
41	12M	MS	90	119	25M	SL	95
43	12M	MS	92	120	25F	SL	102
44	12M	MS	94	123	25F	MS	95
49	13F	ML	82	124	25M	ML	101
51	13M	SS	90	125	25F	SL	97
57	13F	MS	82	129	25M	MS	113
59	13M	MS	80	131	25M	SS	97
60	13F	SL	77				

Appendix V

CLASSIFICATION OF RESPONSES FOR STUDY IIA

Table V-1	Subject Performance on Items in Study IIA
Table V-2	Subjects' Reasons for Correct Responses and Types of Errors in Study IIA
Table V-3	Order in which Items Were Given to Each Subject in Study IIA

Code:
C = Conservation of continuous quantity
D = Measurement with distinguishably different units
I = Measurement with indistinguishably different units
E = Equivalence
1 = Nonequivalence I
2 = Nonequivalence II

Table V-1

Subject Performance on Items in Study IIA

Code: + Correct
0 Incorrect

Subj.	CE	C1	C2	DE	D1	D2	IE	I1	I2	Total correct
4	+	+	+	+	+	+	0	+	0	7
5	0	0	0	0	0	0	0	0	0	0
6	0	0	+	0	0	0	0	0	0	1
7	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	+	+	0	0	0	2
16	0	0	0	0	0	0	0	0	0	0
19	0	+	0	0	+	0	0	+	+	4
21	0	+	0	0	0	0	0	0	0	1
22	+	+	0	+	0	0	0	0	0	3
45	0	+	0	0	0	0	0	0	0	1
46	0	+	+	0	0	0	0	0	0	2
47	0	0	0	0	+	0	+	0	+	3
25	+	0	+	0	0	0	0	0	0	2
27	0	+	0	+	0	0	+	0	0	3
35	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0
38	+	+	0	+	+	0	0	0	0	4
42	0	0	0	0	+	0	0	0	0	1
50	0	+	0	0	+	0	0	0	0	2
52	0	+	0	+	+	0	0	0	0	3
53	+	+	+	+	+	+	0	+	0	7
54	0	0	0	0	+	0	0	0	0	1
55	+	+	0	0	+	+	0	0	0	4
56	0	0	+	+	0	0	0	0	0	2
58	0	0	0	0	0	0	0	0	0	0
61	0	0	0	0	0	0	0	0	0	0
64	0	+	0	0	0	0	0	0	0	1
65	0	0	0	0	0	0	0	0	0	0
71	0	0	0	0	0	+	0	0	+	3
72	0	0	0	0	+	+	+	+	+	5
74	0	0	0	0	0	0	0	0	0	0
75	+	+	+	+	+	+	+	+	+	9
76	0	+	0	+	0	0	0	+	0	3
77	+	+	+	+	+	+	+	+	+	9
78	+	+	+	+	+	+	+	+	+	9
80	0	0	0	0	+	+	0	0	0	2
82	0	+	0	0	+	+	0	0	+	4
86	+	0	+	+	0	0	+	0	0	4

Table V-1 (continued)

Subj.	CE	C1	C2	DE	D1	D2	IE	I1	I2	Total correct
87	+	+	+	+	+	+	+	+	+	9
88	0	+	0	+	+	0	0	+	+	5
90	0	0	0	0	0	0	0	0	0	0
91	+	+	+	+	+	+	+	+	+	9
92	0	0	0	0	0	0	0	0	0	0
93	+	+	+	+	+	+	+	+	+	9
94	+	+	+	+	+	+	0	0	0	6
98	+	+	0	0	0	0	0	0	0	2
100	+	+	+	0	+	0	+	0	0	5
101	+	+	+	+		0	+	0	0	6
107	+	0	+	0	0	0	0	0	0	2
104	0	0	0	0	0	0	0	0	0	0
108	+	+	+	+	+	+	0	+	+	8
110	+	+	+	+	+	+	+	+	+	9
111	+	+	+	+	+	+	+	+	+	9
113	0	+	0	0	+	0	0	+	+	4
116	0	0	0	+	+	+	+	0	0	4
121	+	+	+	+	+	+	+	+	+	9
122	+	+	+	+	+	+	+	+	+	9
126	+	+	+	0	+	+	+	+	+	8
127	0	0	0	0	+	+	0	0	0	2
128	0	0	0	+	+	+	+	+	+	6
130	+	+	+	+	+	+	+	+	+	9

Table V-2

Subjects' Reasons for Correct Response and Types of Errors in Study IIA

Code:
1 Reversibility
2 Statement of operation performed
3 Addition-subtraction
4 Compensation, proportionality
5 Sameness of quantity
6 Reference to previous state
7 No reason given or unclassifiable reason given
8 Height of water or greater number of units
9 Wider container or larger unit

Subj.	CE	C1	C2	DE	D1	D2	IE	I1	I2
4	4	6	6	7	6	6	8	6	8
5	8	8	8	8	8	8	8	8	8
6	8	8	4	8	8	8	8	8	8
7	8	8	8	8	8	8	8	8	8
14	8	8	8	8	6	4	8	8	8
16	8	8	8	8	8	8	8	8	8
19	8	7	8	8	7	8	8	7	7
21	8	7	8	8	8	8	8	8	8
22	6	6	8	6	9	8	8	9	8
45	8	6	8	8	8	8	8	8	8
46	8	7	7	8	8	8	8	8	8
47	8	8	8	8	6	8	6	8	6
25	4	8	6	8	8	8	8	8	8
27	8	6	8	6	8	8	6	8	8
35	8	8	8	8	8	8	8	8	8
40	8	8	8	8	8	8	8	8	8
38	4	6	8	4	4	8	8	8	8
42	8	8	8	8	7	8	8	8	8
50	8	6	8	8	7	8	8	8	8
52	8	4	8	4	4	8	8	8	8
53	4	4	6	6	6	6	8	6	8
54	8	8	8	8	8	8	8	8	8
55	6	6	8	8	4	6	8	8	8
56	8	8	4	7	8	8	8	8	8
58	8	8	8	8	8	8	8	8	8
61	8	8	8	8	8	8	8	8	8
64	8	7	8	8	8	8	8	8	8
65	8	8	8	8	8	8	8	8	8
71	8	8	8	8	4	4	8	8	4
72	8	9	8	8	7	4	7	7	6

Table V-2 (continued)

Subj.	CE	C1	C2	DE	D1	D2	IE	I1	I2
74	8	8	8	8	8	8	8	8	8
75	6	6	6	4	6	6	6	6	6
76	8	7	9	4	8	9	8	7	9
77	4	4	4	6	6	6	6	6	6
78	4	4	6	4	4	4	6	6	6
80	8	8	8	8	6	4	9	8	8
82	8	6	8	8	6	6	8	9	6
86	4	8	4	4	9	9	7	9	8
87	2	6	6	6	6	6	6	6	6
88	8	4	8	6	6	9	8	6	6
90	8	8	8	8	8	8	8	8	8
91	6	4	6	6	6	6	6	6	6
92	8	8	8	8	8	8	8	8	8
93	6	4	6	6	4	6	6	6	8
94	4	4	4	4	4	4	8	9	8
98	4	4	9	9	8	8	8	8	8
100	6	4	6	9	4	8	6	8	8
101	4	4	6	4	6	9	6	8	9
107	6	8	6	8	8	8	8	8	8
104	8	9	8	8	8	8	8	8	8
108	6	4	6	4	4	4	8	6	6
110	6	6	6	6	6	6	6	6	6
111	6	6	6	6	6	6	6	6	6
113	8	4	8	8	6	8	8	7	7
116	8	8	8	6	7	7	6	8	8
121	6	6	6	6	6	6	6	6	6
122	6	6	6	6	6	6	6	6	6
126	6	6	6	6	6	6	6	6	6
127	8	8	8	8	4	4	8	8	8
128	8	8	8	5	7	6	6	6	6
130	6	6	6	6	6	6	6	6	6

Table V-3

Order in which Items Were Given to Each Subject in Study IIA

Subj.	CE	C1	C2	DE	D1	D2	IE	I1	I2
4	4	6	1	2	8	7	5	9	3
5	8	7	3	5	2	1	6	9	4
6	2	8	3	7	5	4	1	9	6
7	9	6	1	3	7	2	5	4	8
14	3	4	9	1	7	8	1	2	5
16	9	1	4	8	7	2	3	6	5
19	7	8	3	2	1	9	6	5	4
21	5	9	7	1	6	4	2	8	3
22	6	9	5	7	3	4	2	1	8
45	1	2	4	8	3	5	9	7	6
46	9	7	2	5	8	4	3	6	1
47	2	7	1	3	6	8	9	4	5
25	8	7	2	3	7	1	4	5	9
27	7	6	5	3	2	8	4	1	9
35	3	6	1	5	4	2	9	8	7
40	7	8	1	5	2	9	4	3	6
38	8	7	5	6	9	4	3	2	1
42	9	5	8	6	3	1	4	1	7
50	5	2	9	4	7	6	1	3	8
52	1	6	4	5	8	2	9	3	7
53	2	4	7	9	1	6	3	8	5
54	3	9	1	7	6	8	5	2	4
55	5	9	2	3	1	8	7	6	4
56	8	4	6	5	9	7	2	1	3
58	4	3	1	6	7	9	8	5	2
61	9	5	3	8	6	1	7	4	2
64	2	3	7	4	8	5	6	9	1
65	4	5	6	9	3	7	2	8	1
71	9	1	3	7	8	5	6	4	2
72	7	3	2	2	5	8	6	4	9
74	4	3	8	6	2	1	9	5	7
75	6	9	8	1	3	4	7	5	2
76	8	5	9	6	4	7	1	2	3
77	4	9	1	2	7	3	6	5	8
78	1	3	4	5	9	6	2	7	8
80	5	1	8	2	6	7	9	4	3
82	5	6	7	1	4	8	3	1	9
86	3	5	6	2	4	7	8	9	1
87	4	3	5	9	6	1	7	8	2
88	1	4	7	8	5	6	2	9	3
90	8	6	4	9	5	3	1	2	7
91	4	3	5	7	2	1	8	9	7

Table V-3 (continued)

Subj.	CE	C1	C2	DE	D1	D2	IE	I1	I2
92	4	1	8	3	9	5	6	2	7
93	5	7	9	3	1	6	8	4	2
94	3	4	8	2	7	6	9	5	1
98	3	4	6	2	5	8	7	9	1
100	5	2	6	1	8	9	7	4	3
101	8	5	3	2	6	4	7	1	9
107	5	1	7	6	3	8	4	2	8
104	1	6	2	5	8	9	4	3	7
108	3	2	4	9	8	5	1	6	7
110	4	1	7	2	3	5	6	9	8
111	3	5	7	2	9	1	4	9	6
113	1	6	3	7	2	9	8	5	4
116	6	5	7	8	4	1	9	3	2
121	4	2	1	7	3	5	6	8	9
122	9	2	1	5	6	8	7	3	4
126	7	8	2	1	3	6	4	5	9
127	4	6	5	9	3	1	7	8	2
128	3	4	7	6	1	2	9	5	8
130	6	4	5	3	1	8	9	8	2

Appendix VI

CLASSIFICATION OF RESPONSES FOR STUDY IIB

Table VI-1 Subject Performance on Items in Study IIB

Table VI-2 Subjects' Reasons for Correct Responses and Types of Errors in Study IIB

Table VI-3 Order in which Items Were Given to Each Subject in Study IIB

Code: D = Measurement with distinguishably different units
I = Measurement with indistinguishably different units
M = Measurement with the same unit into apparent inequality
V = Measurement of unequal appearing quantities with the same unit
E = Equivalence
2 = Nonequivalence II

Table VI-1

Subject Performance on Items in Study IIB

Code: + Correct
0 Incorrect

Subj.	DE	D1	IE	I1	ME	M1	VE	V1	Total correct
1	+	+	+	+	+	+	0	+	7
2	0	0	0	0	0	0	0	+	2
3	0	0	0	0	0	0	+	+	3
8	0	0	0	0	0	0	0	+	2
9	0	0	0	0	0	0	0	+	2
10	0	0	0	0	+	+	+	+	4
11	+	+	0	0	+	+	+	+	6
12	0	0	0	0	+	+	+	+	4
13	0	0	0	0	+	+	+	+	4
15	0	0	0	0	+	+	+	+	4
17	0	0	0	0	+	+	+	+	4
18	0	0	0	0	0	+	0	+	2
20	0	0	0	0	0	0	+	+	2
23	+	+	0	0	+	+	+	+	6
24	0	0	0	0	+	+	+	+	4
48	+	+	+	+	+	+	+	+	8
26	0	0	0	0	+	+	+	+	4
28	0	+	0	+	+	0	+	+	5
29	0	0	0	0	+	+	+	+	4
30	+	+	+	+	+	+	+	+	8
31	+	0	+	0	+	0	+	0	4
32	0	0	0	0	+	+	+	+	4
33	0	+	0	+	0	0	+	0	3
34	0	+	0	+	+	+	+	+	6
36	+	+	+	0	+	0	+	0	5
37	0	0	+	0	+	+	+	+	5
39	0	0	0	0	+	+	+	+	4
41	0	0	0	0	+	+	+	+	4
43	0	0	0	0	0	+	+	+	3
44	0	0	0	0	0	+	+	+	3
49	0	0	0	0	+	+	+	+	4
51	0	0	0	0	+	+	+	+	4
57	+	+	0	+	0	0	+	+	5
59	0	+	0	0	0	0	+	+	3
60	0	0	0	0	+	+	+	+	4
62	+	0	0	0	+	+	+	+	5
63	+	0	0	0	+	+	+	+	5
66	0	0	0	0	0	0	0	0	0

Table VI-1 (continued)

Subj.	DE	D1	IE	I1	ME	M1	VE	V1	Total correct
67	+	+	+	+	0	0	0	0	4
68	0	0	0	0	0	0	0	+	1
69	0	0	0	0	+	+	+	+	4
70	0	0	0	0	+	0	+	+	3
73	0	+	0	0	0	0	+	+	3
79	0	0	0	0	+	+	+	+	4
81	0	0	0	0	+	+	+	+	4
84	+	+	+	+	+	+	+	+	8
85	+	+	+	+	+	+	+	+	8
89	+	+	0	0	+	+	+	+	6
95	+	0	0	0	+	+	+	0	4
99	0	0	0	0	+	+	+	+	4
102	0	0	0	0	0	+	+	+	3
103	0	0	0	0	0	+	+	0	2
106	+	+	0	+	0	+	+	+	6
105	0	0	0	0	+	+	+	0	3
112	0	0	0	0	0	0	+	+	2
114	0	0	0	0	+	+	+	+	4
115	+	+	+	+	+	+	+	+	8
117	+	+	0	0	0	0	+	+	4
118	+	+	0	+	+	+	+	+	7
119	+	+	+	0	+	+	+	+	7
120	0	+	0	0	+	+	+	+	6
123	0	0	0	0	+	+	+	+	4
124	0	0	0	0	+	+	+	+	4

Table VI-2

Subjects' Reasons for Correct Responses and Types of Errors in Study IIB

Code:
1 Reversibility
2 Statement of operation performed
3 Addition-subtraction
4 Compensation, proportionality
5 Sameness of quantity
6 Reference to previous state
7 No reason given or unclassifiable reason given
8 Height of water or greater number of units
9 Wider container, larger unit or same

Subj.	DE	D2	I1	I2	ME	M2	VE	V2
1	6	6	6	6	6	8	6	6
2	8	8	8	8	8	8	6	6
3	8	8	8	8	8	6	6	6
8	8	8	8	8	8	8	6	6
9	8	8	8	8	8	8	6	6
10	8	8	8	8	6	6	6	6
11	4	4	8	8	6	6	6	6
12	8	8	8	8	6	6	6	6
13	8	8	8	8	6	6	2	2
15	8	8	8	8	6	6	6	6
17	8	8	8	8	6	6	6	6
18	8	8	8	8	8	7	8	7
20	8	8	8	8	8	8	6	6
23	7	7	8	8	6	6	6	6
24	8	8	8	9	7	4	7	6
48	7	7	7	7	7	7	7	7
26	8	8	8	8	6	6	6	6
28	8	6	8	7	6	7	6	6
29	8	8	8	8	6	6	6	6
30	6	7	7	7	6	6	6	6
31	7	9	4	9	6	9	6	9
32	8	8	8	8	6	6	6	6
33	9	7	8	7	8	8	6	8
34	8	6	8	6	6	6	6	6
36	6	6	6	9	6	9	6	9
37	8	8	7	8	6	6	6	6
39	8	8	8	8	6	6	6	6
41	8	9	8	8	6	6	6	6
43	8	8	8	8	8	6	6	6
44	8	8	8	8	8	6	6	6
49	8	8	8	8	6	6	6	6
51	8	9	8	8	6	6	6	6

Table VI-2 (continued)

Subj.	DE	D2	IE	I2	ME	M2	VE	V2
57	6	6	8	6	8	8	6	6
59	8	4	8	8	8	8	6	6
60	8	8	8	8	6	6	6	6
62	6	8	8	8	6	6	6	6
63	4	8	8	8	6	6	6	6
66	8	7	8	8	8	8	8	8
67	6	7	6	6	8	8	8	8
68	8	8	8	8	8	8	8	6
69	8	8	8	8	6	6	6	6
70	8	8	8	8	6	8	6	8
73	8	4	8	8	8	8	6	6
79	8	8	8	8	6	6	6	6
81	8	8	8	8	6	6	6	6
84	6	4	6	7	6	6	6	6
85	6	6	6	6	6	6	6	6
89	6	6	8	8	6	6	6	6
95	4	9	8	8	6	4	4	9
96	9	7	8	8	6	6	6	7
97	9	9	8	9	6	8	6	9
99	9	8	8	8	6	6	6	6
102	8	8	8	8	8	6	6	6
103	8	9	8	8	8	6	6	8
106	6	4	8	6	8	6	6	6
105	8	8	8	8	6	6	6	6
112	8	8	8	8	8	8	6	6
114	9	9	8	8	7	4	6	6
115	6	6	6	6	6	6	6	6
117	6	6	8	9	8	9	6	6
118	6	6	8	6	6	6	6	6
119	4	4	7	8	6	6	6	6
120	8	4	8	8	6	6	6	6
123	8	8	8	8	6	6	6	6
124	8	8	8	8	6	6	6	6
125	9	8	8	8	6	8	6	8
129	6	6	8	8	6	6	6	6
131	4	4	8	8	6	6	6	6

Table VI-3

Order in which Items Were Given to Each Subject in Study IIB

Subj.	DE	D2	IE	I2	ME	M2	VE	V2
1	1	7	6	2	4	3	8	5
2	3	6	2	7	1	5	4	8
3	1	8	7	4	5	6	3	2
8	4	5	3	7	6	1	2	8
9	4	7	3	5	1	2	6	8
10	8	7	5	2	3	4	6	1
11	7	8	4	1	6	2	5	3
12	6	4	7	2	1	8	3	5
13	5	7	6	4	3	8	2	1
15	6	5	3	4	8	1	1	7
17	3	8	3	4	8	5	1	2
18	5	2	1	8	7	6	4	6
20	7	6	1	4	3	2	5	8
23	8	5	6	8	1	3	2	4
24	3	6	4	1	8	2	7	5
48	7	5	2	3	8	1	6	4
26	2	3	7	8	6	4	5	1
28	5	6	2	7	4	1	8	1
30	8	3	2	4	1	7	1	6
31	6	7	3	4	1	5	1	8
32	5	6	7	4	2	8	3	1
33	8	4	1	7	3	2	6	5
34	6	7	2	8	3	4	5	1
36	6	7	3	2	1	5	4	8
37	6	3	7	4	2	8	5	1
39	8	7	6	2	5	1	4	3
41	5	4	8	6	3	2	1	7
43	4	6	7	8	3	1	2	5
44	1	7	2	8	6	5	4	3
49	7	2	4	3	6	8	1	5
51	3	8	1	2	7	6	5	4
57	6	5	1	7	3	4	8	2
59	5	7	1	8	3	4	6	2
60	8	6	4	1	3	2	5	7
62	5	3	1	8	6	2	7	5
63	3	5	1	4	6	7	2	8
66	8	4	7	6	1	3	5	2
67	2	1	8	5	3	6	4	7
68	3	6	1	8	2	4	5	7
69	2	3	1	4	7	6	8	5
70	8	3	4	6	7	1	2	5
73	8	1	6	2	3	5	6	4

Table VI-3 (continued)

Subj.	DE	D2	IE	I2	ME	M2	VE	V2
79	8	7	6	1	3	5	2	4
81	8	6	4	2	5	7	3	1
84	3	2	6	1	7	8	4	5
85	3	5	6	2	1	8	4	7
89	4	6	3	7	5	8	1	2
95	5	4	2	1	6	3	6	8
96	6	7	4	3	5	8	2	1
97	1	8	5	6	3	7	2	4
99	1	8	2	5	4	3	6	7
102	5	8	1	6	2	3	4	7
103	8	7	6	5	3	4	1	2
106	7	3	6	5	1	8	2	4
105	6	8	2	5	7	3	4	1
112	8	7	1	5	2	6	4	3
114	3	8	6	1	7	4	5	2
115	5	1	7	8	4	6	3	2
117	7	8	4	5	2	3	6	1
118	8	6	1	7	3	4	2	5
119	4	8	6	5	1	3	7	2
120	8	2	5	4	1	7	3	6
123	7	3	8	2	5	1	4	6
124	8	4	7	5	1	2	3	6
125	5	7	2	1	8	3	6	4
129	6	3	8	1	2	5	7	4
131	7	5	1	2	6	3	4	8

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